AN APPLICATION OF LINEAR PROGRAMMING TO RECRUIT TRANSPORTATION

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Submitted in partial fulfillment of
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ABSTRACT

The problem of transporting recruits from Navy Recruiting Stations to Recruit Training Centers in such a way as to minimize total cost is subjected to solution by standard linear programming methods. A sensitivity analysis of the variation of total cost for different modes of transportation, as influenced by the variation of the proportion of recruits going to each of the two Recruit Training Centers, is carried out. A probabilistic model of the occurrence of a given number of enlistments, up to a fixed quota, at each Recruiting Station is formulated and a simulation of the transportation problem arising is carried out. A rationale is developed for the derivation of a realistic estimator for planning, and some of the difficulties of determining a best estimator are discussed. A computer program for approximating the expected loss for any estimator is presented. Some other types of approach to the problem are indicated.

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1. Introduction.

In today's Navy considerable emphasis is being put on cost reductions of all forms. One important example of an area in which significant cost reductions might be made is that of the transportation of recruits from Navy Recruiting Stations (RS's) to Recruit Training Centers (RTC's). The large number of such "shipments" made indicate that close study of the costs are needed. Accordingly, the Bureau of Naval Personnel, in their letter PERS B-61 KAD of 29 May 1963, requested investigation of methods to reduce these costs. This paper is presented as an illustration of a practical application of the basic linear programming method of solving transportation problems as applied to naval recruits from the time of their enlistment until their arrival at the appropriate RTC.

There are thirty-nine main Naval Recruiting Stations and nineteen class "A" substations, a total of fifty-eight recruit sources. Each RS is assigned, as a quota, a certain percentage of total number of recruits to be included by the Navy Recruiting System for the year. To receive the recruits there are two RTC's each of which is assigned to receive half the total number of recruits enlisted. Recruits can, in principle, be sent from any RS to either RTC by any of three modes of transportation: air, rail, or bus, all of which cost different amounts. In addition, there are differences in cost which are a result of obtaining group rates. In order to analyze the cost of transportation more than superficially, it is necessary to include costs other than those of tickets, e.g., meals, miscellaneous (limousine service, etc.) and pay of recruits during transit. The desired solution of the problem should give the optimum number of recruits to be sent to each RTC from

each RS and should determine the modes by which they should be sent. In addition, since fare rates, cost of living, pay rates, RS quotas, and RTC quotas may all vary, it is necessary to determine the effects of all of these changes on the optimum transportation allocations. Determinations of these effects are known as sensitivity analyses.

This problem, with slight modifications, may be formulated as the linear programming transportation problem. This formulation of the problem is that of the shipment of a homogeneous product from a stated number of sources to a stated (possibly different) number of receiving points (destinations); the amount to be shipped from each source and the amount to be shipped to each destination is predetermined; however, the amount going from a specific source to a specific destination is not. The cost of shipment of one item from any source to any destination is a necessary input. The solution determines the amounts to be shipped from each source to each destination in such a manner as to minimize total cost, and gives the minimum total cost.

In this paper we have made use of a previously written FORTRAN program of the simplex algorithm to solve (on a CDC 1604 computer) the basic transportation problem as posed by the Bureau of Naval Personnel. In addition we have investigated the changes in the solution introduced by varying certain of the constraints (e.g., the restriction that 50% of the men go to each Recruit Training Center). Further, an attempt has been made to come to grips with the more realistic problem in which the number of recruits desired is specified, but the number which will actually enlist can be described only probabilistically.

Since some of the data were unavailable at the time this paper was written, those values were supplied by educated guess. Further,

the actual number of men in the quotas were not known; the percentages of the total were used instead. Thus, this paper is a demonstration of a method which becomes a true solution (within the limits of its assumptions) only when the true values are supplied.

2. Problem Formulation.

In this (classical) statement of the linear programming transportation problem the following information is necessary as input:

It is desired to ship a homogeneous product from each of a given number of destinations in such a manner that each destination <u>receives</u> an exact desired amount (which can be different for each destination). Further, this shipment is to be carried out in such a manner that each origin <u>sends</u> an exact desired amount which can be different for each origin. The total amount that is desired to send is equal to the total amount that is desired to receive. No shipment takes place from destinations to origins. The problem is to be solved in such a way as to minimize the total cost of all this transportation. In other problems some of the above equalities may be inequalities, but in our problem equalities are adequate.

To solve our problem realistically, account must be taken of the fact that seldom will money savings be the only governing criterion in any problem involving people. Hence, it will be necessary to provide solutions to the transportation problem under consideration, which for example, might be to prevent trip length from being excessive. Suppose a decision were made (for reasons of morale, say) that no recruit should spend more than thirty-six hours in travel. Obviously for some RS's the cheapest mode of transportation, which is also the slowest, would no longer be usable, and some costs would be changed. This would raise the total cost of transporting recruits and might well change the pattern of shipments, i.e., how many men are sent between which sources and destinations, for minimizing total cost using the revised costs. Another variation introduced into the problem is that caused by the availability

in some areas of reduced rates when drafts of more than a certain number of men are sent at one time. A solution of the problem using the assumption that advantage is taken of all such available group rates can be justified by assuming that enlistees can be told to report for transportation a long enough time after first enlisting to ensure that the station has enough recruits to obtain group rates. It should be noted that various standard queueing theory models of this situation might be used to select appropriate delays.

The input used in the problems solved for this paper were obtained as follows:

Costs of air, train, and bus tickets from each RS to each RTC (San Diego and Great Lakes) were requested from each RS. In addition, the RS's were asked to supply meal costs, miscellaneous costs, and time enroute for each mode. Not all requests were answered, so missing values were supplied by estimates. In the data supplied, the numbers of men required at the destinations and available at the origins were stated in percents, the information having been supplied by the Bureau of Naval Personnel. The letter prompting this study specified that each Recruit Training Center should receive the same number of recruits, giving the effective determination of that input as fifty percent.

Once having solved the basic problem for the various conditions of restriction on cost (maximum cost, minimum single fare, minimum group fare, minimum with time limit, etc.), it would be of interest to investigate the effect on the optimal solution when changes are made in the system. Since in the transportation problem, the addition of new variables (RS's or RTC's) requires not a variation of the same problem, but the solution of an entire new problem, such additions will not be

investigated. The changes in the system which appear to lead to fruitful investigations are changes in the costs, in the proportion of recruits assigned to each RTC, and in the numbers available at each RS. For a simplified mathematical statement of the foregoing formulation, see Appendix A.

3. Sensitivity Analysis Using Cost Inputs as Parameters.

3.1 The following assumptions are used for this analysis:

First - The statistics presented in the tables are NOT OFFICIAL NAVY statistics and the conclusions from this analysis are strictly the personal opinions of the authors.

Second - It was stated in the introduction of this paper that there are thirty-nine main RS's; however, this analysis was conducted using forty since we believed at the time that Baltimore was a main Recruiting Station. This analysis was completed prior to the discovery of this oversight. This oversight has no effect on the conclusions or the validity of the described method for solving the problem. Hence, for this analysis, recruits are sent from forty RS to two RTC's in accordance with the travel rates or cost coefficients listed in Table I, page 25. These rates are constantly changing; therefore, this table must be "up-dated" to be applicable to a particular time. Computed in these rates are subsistence which includes meals, berthing, limousine service, etc., and pay during travel which amounts to twelve cents per hour.

Third - The overall annual Navy recruiting quota is distributed among the forty Recruiting Stations as listed in Table II, page 26. These percentages were computed by multiplying the annual district quota by the individual Recruiting Station's annual quota. For example:
The first recruiting district is responsible for 16.52% of the overall annual Navy quota. Of this percent, Boston is responsible for 27.00% within the first district; therefore, Boston is responsible for 27.00 times 16.52 or 4.46% of the overall annual Navy quota. These percentage assignments are redistributed slightly each year; therefore, this table also should be "up-dated" to be applicable to a particular year.

Furthermore, some of the RS's have class "A" substations which further breaks the percentage down; however, for the problem discussed in this paper only the forty main recruiting stations were considered as origins because the percentage breakdowns to the class "A" substations was not available. Again, this does not change the basic method of solving the problem.

Fourth - It is emphasized most emphatically that this analysis is based only on costs incurred during transportation and ignores any consideration of costs incurred before arrival of candidates at RS's (i.e., Recruiting Stations operating costs) or after arrival at RTC's (i.e., costs of operating the RTC's).

The sensitivity analysis was conducted by using a fixed set of cost input parameters and varying the percent recruit input to RTC Great Lakes from 0% to 100%. Since the total of 100% recruits is sent to RTC Great Lakes and RTC San Diego, it is evident that RTC San Diego must receive the complement of those recruits sent to RTC Great Lakes. When this approach was used a particular sequence of recruiting stations and corresponding percentages resulted; these percentages were the levels of input to RTC Great Lakes at which the corresponding RS's commenced sending recruits to RTC Great Lakes. On each of the following four pages the parameter of cost input is defined. Below each is tabulated the applicable sequence of RS's.

Max Cost Inputs: Max Cost Inputs are defined as the maximum transportation costs, regardless of mode, between a RS and the two Recruit Training Centers. These inputs were used to put an upper bound on the costs. The use of these cost inputs resulted in transporting the recruits via all three modes; however, each recruit traveled via only a single mode.

Sequence of RS's and the Corresponding Percentage at Which Each RS Commences to Send Recruits to RTC Great Lakes

New York City	1%	Detroit	40%	Raleigh	68%
Milwaukee	8%	Louisville, Ky	44%	Des Moines	70%
Chicago	10%	Columbus	46%	New Orleans	72%
Cleveland	14%	Jacksonville, Fla	47%	Kansas City	73%
Indianapolis	17%	Richmond, Va	50%	Oklahoma City	76%
Baltimore	19%	Boston	51%	Houston	77%
Albany	20%	Nashville	55%	Dallas	79%
Pittsburgh	23%	Cincinnati	58%	Denver	81%
Philadelphia	26%	St. Louis	59%	Albuquerque	83%
Minneapolis	31%	Omaha	62%	Seattle	85%
Ashland, Ky	34%	Little Rock	64%	Portland	87%
Buffalo	36%	Macon, Ga	65%	San Francisco	90%
Washington, DC	38%	Birmingham	66%	Los Angeles	94% - 100%
Columbia, S.C.	39%				

Air Cost Inputs: Air Cost Inputs are defined as the cost to transport all recruits only by air. These inputs were used primarily to arrive at a cost figure when minimum time enroute is the primary consideration. For Chicago, where no air transportation is available to RTC Great Lakes, the train fare was used under the assumption that time spend enroute was less than by bus.

Sequence of RS's and the Corresponding Percentage at Which Each RS Commences to Send Recruits to RTC Great Lakes

Albany	1%	Indianapolis	44%	New Orleans	69%
Louisville	3%	Baltimore	45%	Omaha	70%
Cleveland	4%	Columbus	47%	Kansas City	72%
New York City	7%	Nashville	48%	Little Rock	74%
Boston	15%	Cincinnati	50%	Oklahoma City	76%
Ashland	19%	Raleigh	52%	Dallas	77%
Richmond	21%	Columbia	53%	Houston	79%
Philadelphia	22%	Jacksonville	55%	Denver	81%
Detroit	27%	St. Louis	57%	Albuquerque	83%
Milwaukee	31%	Macon	60%	Seattle	85%
Washington, DC	33%	Birmingham	61%	Portland	87%
Chicago	34%	Des Moines	63%	San Francisco	90%
Pittsburgh	38%	Minneapolis	65%	Los Angeles	94% - 100%
Buffalo	42%				

<u>MinT Cost Inputs</u>: MinT Cost Inputs are defined as the minimum costs allowing for a maximum time enroute of thirty-six hours. Thirty-six hours is an arbitrary value assumed to be realistic by the authors and any other value might be used to obtain comparable results. With these cost inputs, as with the Max cost inputs, the transporting of recruits via all three modes resulted.

Sequence of RS's and the Corresponding Percentage at Which Each RS Commences to Send Recruits to RTC Great Lakes

Ashland, Ky	1%	Washington, DC	41%	Little Rock	69%
Boston, Mass	2%	Macon	42%	New Orleans	70%
Louisville, Ky	6%	Indianapolis	43%	Omaha	71%
Richmond	8%	Buffalo	45%	Kansas City	73%
Philadelphia	9%	Chicago	47%	Denver	76%
New York City	14%	Burmingham	52%	Oklahoma City	77%
Albany	21%	Cincinnati	54%	Dallas	79%
Cleveland	24%	Jacksonville	55%	Houston	81%
Detroit	27%	St. Louis	58%	Albuquerque	83%
Baltimore	31%	Raleigh	61%	Portland	85%
Milwaukee	33%	Columbia	62%	San Francisco	87%
Columbus	35%	Des Moines	63%	Seattle	92%
Nashville	36%	Minneapolis	65%	Los Angeles	94% - 100%
Pittsburgh	38%				

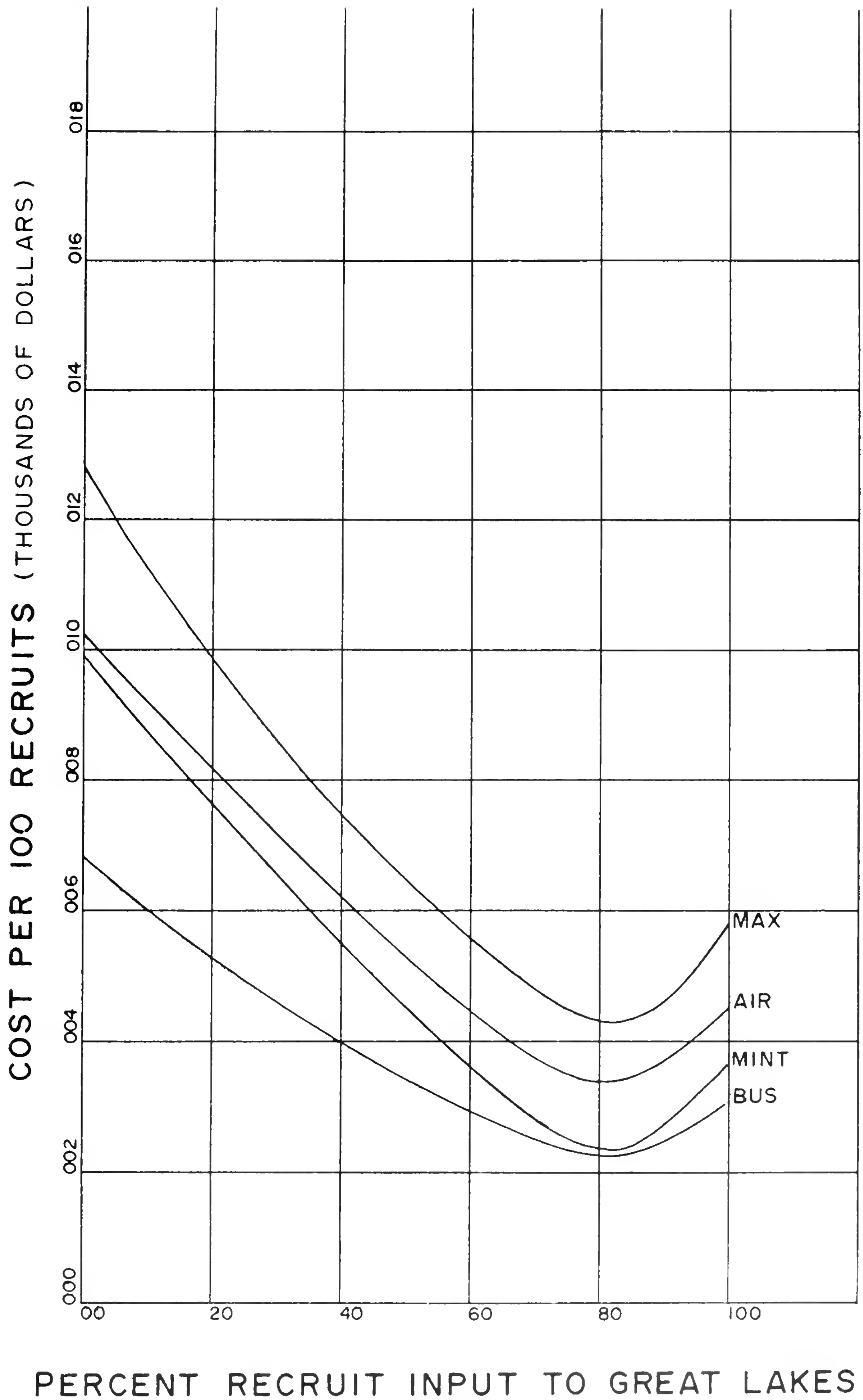
Bus Cost Inputs: Bus Cost Inputs are defined as the costs to transport all recruits by bus. No maximum time enroute was considered with these inputs. We realize that it is not feasible to order a recruit to ride a bus for seventy-eight hours from Portland, Maine to RTC San Diego, however, these inputs also correspond to the minimum costs available and thus, put a lower bound on the costs.

Sequence of RS's and the Corresponding Percentage at Which Each RS Commences to Send Recruits to RTC Great Lakes

New York City	1%	Columbus	44%	Birmingham	68%
Philadelphia	8%	Ashland	46%	Omaha	70%
Milwaukee	13%	Louisville	47%	New Orleans	72%
Chicago	15%	Macon	49%	Kansas City	73%
Pittsburgh	19%	Buffalo	50%	Oklahoma City	76%
Baltimore	22%	Columbia	52%	Dallas	77%
Indianapolis	24%	St. Louis	54%	Houston	79%
Detroit	25%	Raleigh	56%	Denver	81%
Cleveland	30%	Albany	58%	Albuquerque	83%
Minneapolis	33%	Des Moines	60%	Portland	85%
Washington, DC	36%	Nashville	62%	Seattle	87%
Richmond	37%	Jacksonville	64%	San Francisco	90%
Cincinnati	38%	Little Rock	67%	Los Angeles	94% - 100%
Boston	40%				

3.2 A graph of percent recruit input to RTC Great Lakes vs. transportation costs per one-hundred recruits is given in Figure I, page 14. This graph illustrates how the variation of recruit input between the two RTC's affects transportation costs. The curves on the graph were plotted by using the computed data found in Tables III and IV, pages 27 and 28 respectively.

3.3 A FORTRAN listing of the linear program used for the previous sensitivity analysis is given in Appendix E. This listing is included as an aid for those who desire to pursue this transportation problem further; a detailed explanation of this program is not considered essential to the results of this paper. A format of the computer print-out of the above program for a 5% interval, starting at 30% recruit input to RTC Great Lakes, and using 1% increments and Air cost inputs is given in Tables VI-A through VI-F, pages 30 to 35. Table V, page 29, presents a number code which correlates the P-vectors in Tables VI-A through VI-F, pages 30 to 35, to their respective RS's and RTC's.


COST PER 100 RECRUITS (THOUSANDS OF DOLLARS)
000
002
004
006
008
010
012
014
016
018
00
20
40
60
80
100
MAX
AIR
MINT
BUS
PERCENT RECRUIT INPUT TO GREAT LAKES

4. Statistical Variation.

In the previous formulation of our problem, the basic assumption was that the inputs to the problem were known perfectly, but it is quite likely that this will not be the case. Indeed, it is apparent that there is a positive probability that the number of men enlisted at any RS will not be equal to that RS's assigned quota. In other words, the number of men to be transported from any RS may be regarded as a random variable. Regarding this random variable, β, as a parameter to the problem, we note that Garvin [3] points out that for <u>any</u> parameter, B, whose probability density (or mass) function is known, the minimum of the objective function, z_o, obeys the relation

$$E\left[z_o\ (B)\right] \leq z_o\ (E\left[B\right]) \tag{4.1}$$

where:

$E\left[B\right]$ is the vector of expected values of the β's.

$z_o\ (B) = \min z(b)$, the minimum objective function expressed as a function of B.

The lack of equality above makes the problem of saying something meaningful about z more difficult.

Even if we could easily find the value in the left side of equation (4.1), we would have an expression only for an ideal situation which cannot exist in the real world. In fact what we wish to know is what policy we should choose in order to approach this ideal more closely than by the use of any other policy. One approach to such problems is to find an estimator for the parameter B, and then to use this estimator, $\hat{B}$, to define our policy.

In attempting to find the optimum estimator, $\hat{B}$, we have made certain assumptions concerning the uncertain (i.e., statistically varying) value of B, which is a vector representing the number of enlistments at each RS plus the number of enlistees going to one of the RTC's. These assumptions and the models arising therefrom are discussed fully in Appendix B.

It was found (see Appendix B) that there is no simple analytical solution to the problem of determining a best estimator. The problem of even evaluating any given estimator by analytical means is so great as to be impractical to solve; so the authors have taken the approach of using the same assumptions in devising a Monte Carlo simulation. As discussed more fully in Appendix B, it is assumed that the number of people arriving at a recruiting station follow the Poisson probability law and that the number of those who actually enlist is governed by the binomial probability law. In our simulation, this random amount is determined for each RS and the problem solved to find the least cost for that set of inputs. The program is designed to solve any desired number of such subproblems in order to find an approximation to the value of the left side of equation (4.1).

The next assumption we make is that there is, conceptually, some loss in not solving the sub-problems this way, and that by following a fixed policy we will have to incur this loss at least part of the time. By the use of an estimator we can find a policy. The policy is the transportation pattern developed by solution to a sub-problem in which the estimator that we choose is used as the input. When this policy is applied to the sample of sub-problems we obtain, for each sub-problem, a difference in cost which will be either zero or more

and which is the loss for that particular sub-problem. In our Monte Carlo simulation we have evaluated only one estimator, the average B. Of course, given any other estimator, we could evaluate it also. Having found the losses we can find the average loss, which is termed sample risk. We can also find the sample deviation of loss, thereby gaining some idea of how much any particular loss may vary from this average. With larger and larger numbers of sub-problems we can more closely approach the "true" values of these statistics. The results of the Monte Carlo simulation are discussed in section 5.3.

Another possible estimator which might be used under certain circumstances is discussed in Appendix D.

5. Results and Discussion.

5.1 It is interesting to observe that the sequence of the RS's changed when the cost inputs were changed for some reason, such as changing the mode of travel (air to bus), or changing the criterion for choosing a particular mode of travel (limited time enroute).

Using Max Cost Inputs, it is noted that when the input to RTC Great Lakes is changed to 1%, New York is the first RS to send recruits to RTC Great Lakes, and it sends 1% of its quota. New York's total quota is 7.35%; therefore, as the input to RTC Great Lakes is increased by 1% increments, New York increases its quota by 1% increments until its total quota is sent there. One explanation for this could be that New York is responsible for the largest overall Navy quota. However, referring to Table I, page 25, it is observed that the DIFFERENCE in Max costs for New York is \$211.02 - \$44.96 = \$166.06. This difference is larger than for any other city; therefore, it would seem that this combination would give the greater savings.

As the recruit input to RTC Great Lakes continues to change by 1% increments it is noted that when the input reaches 8%, Milwaukee is the next RS to change its distribution and commence sending recruits to RTC Great Lakes. Why should Milwaukee send recruits to RTC Great Lakes before Chicago? Once again it is observed that the DIFFERENCE in Max costs for Milwaukee is \$152.99 - \$10.09 = \$142.90, compared to \$143.39 - \$1.37 = \$142.02, for Chicago. It is interesting to note that the input to RTC Great Lakes must reach 10% before Chicago starts sending recruits to Great Lakes. Although Chicago is the closest recruiting station to Great Lakes, this does not imply that the minimum distance traveled is going to give maximum savings.

Again referring to Table I, page 25, only this time using the Air or Bus costs, it is observed that the first RS to commence sending recruits to both RTC's is not New York, but that city that has the greatest DIFFERENCE in the appropriate costs. When that RS has commenced sending its total quota to RTC Great Lakes, the next city with the greatest DIFFERENCE in costs commences to send recruits to both RTC's. This method continues until all RS's are sending their total quota to RTC Great Lakes. Therefore, when the problem is linear and involves two destinations, as in this problem, we see that it is the DIFFERENCE in costs of the applicable mode of travel to the two destinations that is the controlling factor.

5.2 The graph in Figure 1, page 14, illustrates that there is a decrease in travel costs up to an 82% input of recruits to RTC Great Lakes and then the costs begin to increase. In the case of Max and MinT cost inputs, where it is recalled that mixed travel modes were used, the minimum occurred at roughly the same percentage. Thus, the decrease occurs regardless of the mode of transportation.

Why should the minimum costs occur at such a high percentage input to RTC Great Lakes? It can be conjectured that one possible explanation might be:

1. It costs less to transport recruits from all parts of the country to RTC Great Lakes than to RTC San Diego because RTC Great Lakes is geographically more centrally located.
2. The more populous areas are responsible for a larger overall annual recruiting quota and the majority of these areas are located nearer RTC Great Lakes than RTC San Diego.

We would like to emphasize at this point that we have discussed only that the transportation costs _to_ RTC Great Lakes are less than those _to_ RTC San Diego up to a certain percentage of recruit input. We have made _no_ cost analyses of sending recruits _through_ the two Recruit Training Centers.

5.3 In testing the Monte Carlo approach to this problem, first a sample of fifty randomly generated B-vectors were analyzed. The inputs to the random vector generator sub-program were,

1) RS quotas expressed as integer numbers,
2) the average number of interviews held per day at each RS, and
3) the individual probability of success (obtaining an enlistment) per interview for each RS.

Values of the last two parameters were quite arbitrarily chosen, the average interview number being made approximately a fourth more than the annual quota divided by 365. The probability of success at an interview was chosen arbitrarily over a range from 55% to 80%. A table of these inputs is given in Table VII-B. The vectors resulting had entries of a fair number of stations (about half in most cases) which fulfilled their quota. A typical vector resulting is shown in Table VII-C. The other percent-of-quotas-enlisted ranged down to 64%, but the total quota was filled to a level never less than 93% and never as great as 95% in the fifty samples. Further, the solution pattern of transportation was the same for each random vector, _for these arbitrary parameter values_. Typical results of the solution of one vector are given in Table VII-D. In short, the solution was quite insensitive to changes generated in a small sample by these parameter values, and the sample deviation of the random variable β for the RTC was small. Because

of this result, it was decided that more runs would be desirable to determine whether different parameter values might lead to very different results. Accordingly, four samples were computed, with fifteen random vectors each and with combinations of parameters that differed greatly. The average number of interview for each RS was given the value of the quota of that RS divided by 365 for two runs and twice that much for two others. The probability of interview success was fixed by 55% for all stations for two runs and 65% for the other two runs. These were, of course, combined in such a way as to get all four combinations.

Finally, our simulation also produced sample mean loss, an approximation to the risk for the given estimator, and the sample deviation of that loss. Examples of these results are included in Table VII-D. When low averages and low probabilities (55%) were used, the percent of total quota filled never dropped below 56% and never reached 57%. Individual percent of quotas ranged from 39% to 73%. When low averages and high probabilities (75%) were used the percent of total quota filled ranged between 75% and 77% while individual quotas were filled to levels from 55% to 96%. For the combination of high average number of arrivals and low probability of enlistments total quota fulfillment never fell below 99% while individual station quotas were filled to levels as low as 80% for the very few in each sample not fulfilling their quotas. The combination of high probability and high average gave 100% fulfillment for all fifteen samples.

The lack of variation in the percent of total quota filled can be viewed as a substantiation of the validity of the statement that for large samples β_{m+1} can be approximated by a normal random variable <u>independent of the other m</u> random variables.

This lack of variation had one other notable effect. The mean B vector, $\bar{\bar{B}}$, proved to be a very good estimator; there were, in fact, no samples on which the result was non-optimal. It should be noted that should the use of the transformation $\bar{\bar{P}}^{-1}$ result in a sample with a non-optimal solution, our program will compute the loss on that sample. Therefore, not only was $\bar{\bar{B}}$ a good estimator <u>for our selected inputs</u>, but it is also obvious that <u>for these inputs</u>, a static model is adequate. Of course, use of different quotas and different parameters might cause the probability mass functions governing the inputs to "spread" so much that non-optimal solutions might result in enough cases to make a dynamic model desirable.

5.4 In the probability model developed herein many simplifying assumptions were made which might have an effect on the conclusions. Some of them are discussed below.

1. It was assumed that quotas assigned to RS's were fixed. It has been learned that such quotas may in fact be changed during the year in order to capitalize on peculiar seasonal or regional situations effecting recruitment.
2. It was assumed that the number of interviews held by a recruiting station could be described by a Poisson probability mass function. In using the Poisson distribution, one assumes that the probability of more than one event occurring in time Δt is of smaller order than Δt so that as Δt approaches zero the probability of two or more events occurring in Δt also is negligible. In practice recruiters often give talks to large groups of high school or college students. This might

be said to be more than one interview; however, it is believed that the prospective enlistees must still enter a queue of some sort before making a commitment to enlist, so that the mass lecture may be regarded as one of the factors determining α, the average number of interviews for the j^{th} RS rather than as an actual multiple interview.

3. Probabilistic independence of the occurrence of interviews was assumed. Arguments contrary to this assumption have been acknowledged above and some defense given. As a further defense, we note that the amount of personal effect required of the recruiter is a factor tending to promote the statistical independence of these random variables.

4. It has been assumed that the parameters of the Poisson and binomial distributions used could be determined for each RS. They cannot be completely determined, of course, but it is likely that enough data can be obtained from RS records to arrive at very good estimates.

Although some of these objections are forceful, none is believed to be strong enough to invalidate the usefulness of the results.

6. Conclusions.

It can be concluded from this investigation that:

1. We have been able to devise a method enabling us to simulate (at least approximately) the uncertainties caused by the probabilistic nature of numbers of enlistments and thus, can consider our model to be more than a mere laboratory affair.
2. We have been able to develop a rationale for the derivation of an estimator and have been able to obtain an approximation of the success we may expect in solving our problem relative to an ideal solution, given any particular estimator.
3. By means of simulation, we have found that the mean input vector, $\bar{\bar{B}}$, is an estimator which yields optimal solutions for a very large number of samples and for widely varying parameters. We have seen that this indicates the adequacy of a static model.
4. We have indicated another promising estimator that might be developed that could have considerable appeal to a planner under certain circumstances.
5. We have been able to devise a method allowing us to present to the planner an assessment of the cost of certain decisions, namely the choice of assigning various quotas to each of the two RTC's, the choice of putting a limit on travel time, and the choice of an estimator.
6. We have found the simple principle underlying the choice of routes to be used by the RS's, that of maximum cost DIFFERENCE.

TABLE I

COST COEFFICIENTS FROM RECRUITING STATIONS TO RECRUIT TRAINING CENTERS

GLAKES	SDIEGO	GLAKES	SDIEGO	GLAKES	SDIEGO	GLAKES	SDIEGO	GLAKES	SDIEGO	
ALBANY		ALBUQUERQUE		ASHLAND		BALTIMORE		BIRMINGHAM		
52.23	161.92	69.82	40.09	37.14	138.40	39.33	133.42	42.11	118.44	AIR
74.18	203.36	76.59	55.77	27.56	149.27	79.97	214.65	37.73	108.94	TRAIN
52.45	98.12	42.72	31.31	16.70	70.82	28.08	98.01	22.60	60.37	BUS
BOSTON		BUFFALO		CHICAGO		CINCINNATI		CLEVELAND		
52.74	154.46	24.98	119.55	0.00	97.17	20.48	106.56	22.37	126.16	AIR
86.83	176.98	36.87	145.30	1.37	143.39	30.45	119.93	21.96	164.16	TRAIN
34.27	90.20	23.13	74.90	1.28	73.77	11.55	68.40	17.25	82.80	BUS
COLUMBIA		COLUMBUS		DALLAS		DENVER		DES MOINES		
43.99	128.56	24.84	117.29	54.31	76.11	55.76	57.96	24.26	99.05	AIR
59.34	164.84	31.94	128.69	61.61	64.87	51.28	68.71	30.31	104.84	TRAIN
34.13	84.35	12.73	68.40	33.25	47.18	27.53	30.62	15.04	60.26	BUS
DETROIT		HOUSTON		INDIANAPOLIS		JACKSONVILLE		KANSAS CITY		
22.49	122.92	66.41	76.91	14.78	109.09	49.26	130.35	24.22	79.71	AIR
17.13	94.88	71.18	102.16	15.80	152.55	59.53	156.11	26.78	96.35	TRAIN
14.65	82.46	41.08	53.27	11.70	79.55	38.63	81.82	18.72	54.56	BUS
LITTLE ROCK		LOS ANGELES		LOUISVILLE		MACON		MILWAUKEE		
43.94	91.55	97.81	10.09	20.78	129.95	50.77	128.44	10.09	107.10	AIR
33.27	124.00	138.74	4.52	30.78	118.89	87.66	165.41	3.35	152.99	TRAIN
20.37	59.78	71.55	3.56	11.79	65.55	30.97	83.10	2.23	77.44	BUS
MINNEAPOLIS		NASHVILLE		NEW ORLEANS		NEW YORK		OKLAHOMA CITY		
25.24	95.70	26.53	118.18	44.00	103.02	44.96	147.51	43.43	77.22	AIR
25.92	141.94	28.55	109.04	59.59	131.94	42.96	211.02	41.10	66.38	TRAIN
13.92	76.01	15.50	60.01	35.14	72.10	37.23	124.88	29.52	57.35	BUS
OMAHA		PHILADELPHIA		PITTSBURGH		PORTLAND		RALEIGH		
23.74	86.19	41.32	142.01	29.63	124.72	107.28	61.63	47.09	131.87	AIR
33.64	113.93	76.78	204.05	51.19	178.49	78.41	38.53	62.00	137.01	TRAIN
19.00	56.63	31.21	110.60	22.54	94.05	68.27	31.50	29.76	76.86	BUS
RICHMOND		SAN FRANCISCO		SEATTLE		ST LOUIS		WASHINGTON		
41.62	142.34	91.55	20.98	106.29	62.97	19.49	98.49	34.31	130.74	AIR
63.69	156.70	128.32	27.80	103.95	72.75	13.44	100.15	69.79	177.98	TRAIN
29.93	89.80	63.54	14.38	60.65	23.36	8.81	56.70	31.31	93.25	BUS

TABLE II

DISTRIBUTION OF OVERALL ANNUAL RECRUITING QUOTA

ALBANY	2.56	DES MOINES	1.86	NEW ORLEANS	1.24
ALBUQUERQUE	1.43	DETROIT	4.33	NEW YORK	7.35
ASHLAND	1.36	HOUSTON	2.53	OKLAHOMA CITY	1.41
BALTIMORE	1.39	INDIANAPOLIS	1.66	OMAHA	1.67
BIRMINGHAM	2.05	JACKSONVILLE	2.44	PHILADELPHIA	4.88
BOSTON	4.46	KANSAS CITY	2.50	PITTSBURGH	3.06
BUFFALO	2.14	LITTLE ROCK	1.36	PORTLAND	2.14
CHICAGO	4.56	LOS ANGELES	6.20	RALEIGH	1.47
CINCINNATI	1.67	LOUISVILLE	1.39	RICHMOND	1.39
CLEVELAND	2.92	MACON	1.35	SAN FRANCISCO	4.80
COLUMBIA	1.47	MILWAUKEE	1.92	SEATTLE	2.75
COLUMBUS	1.39	MINNEAPOLIS	3.38	ST LOUIS	2.77
DALLAS	1.83	NASHVILLE	2.05	WASHINGTON	0.83
DENVER	1.80				

TABLE III

TABULATION OF MAXIMUM TRAVEL COSTS PER RECRUIT AND CORRESPONDING PERCENTAGE INPUT TO GREAT LAKES

00	128.69	26	91.04	52	63.18	78	43.76
01	127.03	27	89.76	53	62.28	79	43.45
02	125.37	28	88.49	54	61.38	80	43.31
03	123.71	29	87.22	55	60.48	81	43.16
04	122.05	30	85.95	56	59.58	82	43.03
05	120.39	31	84.75	57	58.69	83	43.01
06	118.73	32	83.59	58	57.79	84	43.22
07	117.07	33	82.43	59	56.92	85	43.54
08	115.56	34	81.28	60	56.11	86	43.88
09	114.13	35	80.16	61	55.31	87	44.23
10	112.70	36	79.07	62	54.50	88	44.69
11	111.28	37	77.99	63	53.70	89	45.15
12	109.86	38	76.90	64	52.99	90	46.15
13	108.44	39	75.85	65	52.11	91	47.16
14	107.02	40	74.81	66	51.33	92	48.16
15	105.61	41	73.81	67	50.57	93	49.17
16	104.19	42	72.81	68	49.81	94	50.23
17	102.78	43	71.80	69	49.06	95	51.51
18	101.41	44	70.80	70	48.31	96	52.80
19	100.06	45	69.81	71	47.57	97	54.09
20	98.72	46	68.83	72	46.84	98	55.37
21	97.43	47	67.87	73	46.13	99	56.66
22	96.14	48	66.90	74	45.43	100	57.95
23	94.86	49	65.93	75	44.74		
24	93.58	50	65.00	76	44.39		
25	92.31	51	64.08	77	44 07		

TABULATION OF AIR TRAVEL COSTS PER RECRUIT AND CORRESPONDING PERCENTAGE INPUT TO GREAT LAKES

00	102.83	26	76.00	52	51.31	78	34.18
01	101.74	27	75.00	53	50.46	79	34.04
02	100.64	28	74.00	54	49.62	80	33.94
03	99.54	29	72.99	55	48.79	81	33.84
04	98.45	30	71.99	56	47.98	82	33.82
05	97.42	31	70.99	57	47.18	83	33.90
06	96.38	32	70.02	58	46.39	84	34.20
07	95.34	33	69.05	59	45.60	85	34.62
08	94.32	34	68.09	60	44.81	86	35.05
09	93.29	35	67.14	61	44.03	87	35.49
10	92.27	36	66.18	62	43.27	88	35.95
11	91.24	37	65.22	63	42.51	89	36.40
12	90.21	38	64.26	64	41.76	90	37.11
13	89.19	39	63.31	65	41.02	91	37.81
14	88.16	40	62.36	66	40.31	92	38.51
15	87.14	41	61.41	67	39.61	93	39.22
16	86.12	42	60.46	68	38.90	94	39.96
17	85.10	43	59.52	69	38.29	95	40.83
18	84.08	44	58.57	70	37.71	96	41.71
19	83.06	45	57.63	71	37.13	97	42.59
20	82.05	46	56.69	72	36.57	98	43.46
21	81.04	47	55.76	73	36.02	99	44.34
22	80.03	48	54.84	74	35.49	100	45.22
23	79.02	49	53.92	75	35.01		
24	78.02	50	53.02	76	34.67		
25	77.01	51	52.16	77	34.40		

TABLE IV

TABULATION OF MINT TRAVEL COSTS PER RECRUIT AND CORRESPONDING PERCENTAGE INPUT TO GREAT LAKES

00	99.43	26	70.05	52	43.69	78	24.10
01	98.22	27	68.96	53	42.73	79	23.92
02	97.01	28	67.83	54	41.78	80	23.78
03	95.81	29	66.80	55	40.84	81	23.67
04	94.61	30	65.72	56	39.92	82	23.57
05	93.40	31	64.64	57	39.00	83	23.54
06	92.20	32	63.59	58	38.10	84	23.65
07	91.02	33	62.54	59	37.20	85	24.08
08	89.89	34	61.49	60	36.31	86	24.55
09	88.77	35	60.44	61	35.46	87	25.24
10	87.66	36	59.41	62	34.61	88	26.00
11	86.55	37	58.38	63	33.76	89	26.77
12	85.44	38	57.36	64	32.92	90	27.53
13	84.34	39	56.34	65	32.09	91	28.30
14	83.23	40	55.32	66	31.27	92	29.13
15	82.13	41	54.31	67	30.45	93	29.95
16	81.03	42	53.33	68	29.63	94	30.31
17	79.92	43	52.35	69	28.90	95	31.75
18	78.82	44	51.38	70	28.20	96	32.70
19	77.72	45	50.41	71	27.52	97	33.64
20	76.61	46	49.45	72	26.85	98	34.58
21	75.51	47	48.48	73	26.21	99	35.53
22	74.42	48	47.52	74	25.60	100	36.47
23	73.32	49	46.57	75	24.99		
24	72.23	50	45.61	76	24.67		
25	71.14	51	44.65	77	24.37		

TABULATION OF BUS TRAVEL COSTS PER RECRUIT AND CORRESPONDING PERCENTAGE INPUT TO GREAT LAKES

00	68.75	26	48.60	52	33.29	78	22.87
01	67.88	27	47.92	53	32.79	79	22.74
02	67.00	28	47.24	54	32.31	80	22.62
03	66.12	29	46.56	55	31.83	81	22.51
04	65.25	30	45.90	56	31.36	82	22.48
05	64.37	31	45.25	57	30.89	83	22.50
06	63.49	32	44.59	58	30.43	84	22.61
07	62.62	33	43.97	59	29.97	85	22.95
08	61.79	34	43.35	60	29.52	86	23.32
09	61.00	35	42.73	61	29.06	87	23.69
10	60.21	36	42.11	62	28.61	88	24.07
11	59.41	37	41.50	63	28.17	89	24.44
12	58.62	38	40.91	64	27.73	90	24.92
13	57.86	39	40.34	65	27.29	91	25.41
14	57.11	40	39.78	66	26.86	92	25.90
15	56.38	41	39.22	67	26.46	93	26.38
16	55.65	42	38.66	68	26.07	94	26.91
17	54.93	43	38.10	69	25.70	95	27.59
18	54.20	44	37.54	70	25.32	96	28.27
19	53.48	45	36.99	71	24.94	97	28.95
20	52.76	46	36.44	72	24.57	98	29.62
21	52.05	47	35.90	73	24.21	99	30.30
22	51.34	48	35.36	74	23.85	100	30.98
23	50.64	49	34.84	75	23.49		
24	49.96	50	34.32	76	23.21		
25	49.28	51	33.80	77	23.01		

TABLE V

NUMBER CODE OF RECRUITING STATIONS AND RECRUIT TRAINING CENTERS

RECRUITING STATION	GLAKES	SDIEGO
ALBANY	P1	P2
BOSTON	P3	P4
BUFFALO	P5	P6
NEW YORK	P7	P8
ASHLAND	P9	P10
BALTIMORE	P11	P12
LOUISVILLE	P13	P14
RICHMOND	P15	P16
WASHINGTON	P17	P18
BIRMINGHAM	P19	P20
COLUMBIA	P21	P22
JACKSONVILLE	P23	P24
MACON	P25	P26
NASHVILLE	P27	P28
RALEIGH	P29	P30
CINCINNATI	P31	P32
CLEVELAND	P33	P34
COLUMBUS	P35	P36
PHILADELPHIA	P37	P38
PITTSBURGH	P39	P40
CHICAGO	P41	P42
DETROIT	P43	P44
INDIANAPOLIS	P45	P46
MILWAUKEE	P47	P48
ST LOUIS	P49	P50
DENVER	P51	P52
DES MOINES	P53	P54
KANSAS CITY	P55	P56
MINNEAPOLIS	P57	P58
OMAHA	P59	P60
ALBUQUERQUE	P61	P62
DALLAS	P63	P64
HOUSTON	P65	P66
LITTLE ROCK	P67	P68
NEW ORLEANS	P69	P70
OKLAHOMA CITY	P71	P72
LOS ANGELES	P73	P74
PORTLAND	P75	P76
SAN FRANCISCO	P77	P78
SEATTLE	P79	P80

TABLE VI-A

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .719862E+04

GREAT LAKES PERCENT .300E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(6)	.214760E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(43)	.365571E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(18)	.830700E+00
P(37)	.488250E+01
P(22)	.147832E+01
P(24)	.244575E+01
P(26)	.135875E+01
P(28)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(20)	.205443E+01
P(36)	.139500E+01
P(12)	.139941E+01
P(40)	.306900E+01
P(42)	.456274E+01
P(44)	.682690E+00
P(46)	.166334E+01
P(48)	.192276E+01
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(33)	.292950E+01

TABLE VI-B

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .709928E+04

GREAT LAKES PERCENT .310E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(6)	.214760E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(12)	.139941E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(47)	.317310E+00
P(33)	.292950E+01
P(22)	.147832E+01
P(37)	.488250E+01
P(26)	.135875E+01
P(20)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(24)	.244575E+01
P(36)	.139500E+01
P(28)	.205443E+01
P(40)	.306900E+01
P(42)	.456274E+01
P(18)	.830700E+00
P(46)	.166334E+01
P(48)	.160545E+01
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(43)	.433840E+01

TABLE VI-C

PRJBLEM. RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .700227E+04

GREAT LAKES PERCENT .320E+02

BASIS VECTORS AND COEFFICIENTS

VECTJR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(47)	.131731E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(12)	.139941E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(18)	.830700E+00
P(33)	.292950E+01
P(22)	.147832E+01
P(24)	.244575E+01
P(37)	.488250E+01
P(20)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(26)	.135875E+01
P(36)	.139500E+01
P(28)	.205443E+01
P(40)	.306900E+01
P(42)	.456274E+01
P(6)	.214760E+01
P(46)	.166334E+01
P(48)	.605450E+00
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(43)	.433840E+01

TABLE VI-D

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .690549E+04

GREAT LAKES PERCENT .330E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(47)	.192276E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(12)	.139941E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(17)	.394550E+00
P(20)	.205443E+01
P(43)	.433840E+01
P(24)	.244575E+01
P(37)	.488250E+01
P(28)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(26)	.135875E+01
P(36)	.139500E+01
P(22)	.147832E+01
P(40)	.306900E+01
P(42)	.456274E+01
P(6)	.214760E+01
P(46)	.166334E+01
P(18)	.436150E+00
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(33)	.292950E+01

TABLE VI-E

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .680941E+04

GREAT LAKES PERCENT .340E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(41)	.563850E+00
P(7)	.735140E+01
P(9)	.136107E+01
P(47)	.192276E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(17)	.830700E+00
P(20)	.205443E+01
P(22)	.147832E+01
P(24)	.244575E+01
P(37)	.488250E+01
P(28)	.205443E+01
P(43)	.433840E+01
P(32)	.167400E+01
P(26)	.135875E+01
P(36)	.139500E+01
P(30)	.147832E+01
P(40)	.306900E+01
P(42)	.399889E+01
P(12)	.139941E+01
P(46)	.166334E+01
P(6)	.214760E+01
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(33)	.292950E+01

TABLE VI-F

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .671361E+04

GREAT LAKES PERCENT .350E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(41)	.156385E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(47)	.192276E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(17)	.830700E+00
P(37)	.488250E+01
P(43)	.433840E+01
P(24)	.244575E+01
P(26)	.135875E+01
P(28)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(20)	.205443E+01
P(36)	.139500E+01
P(22)	.147832E+01
P(40)	.306900E+01
P(42)	.299889E+01
P(12)	.139941E+01
P(46)	.166334E+01
P(6)	.214760E+01
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(33)	.292950E+01

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APPENDIX A

BASIC SYMBOLOGY OF THE TRANSPORTATION PROBLEM

It is desired to ship a homogeneous product in integral amounts, a_i $(i = 1,\ldots m)$ from each of $\underline{m}$ origins and to have it arrive at $\underline{n}$ destinations in such a manner that each destination receives an amount, b_j $(j - 1,\ldots n)$. The cost c_{ij} of sending a unit amount from the i^{th} origin to the j^{th} destination is known for all ij combinations. Here the total to be sent equals the total demanded, i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

The problem is to select the amounts x_{ij} to be shipped from the i^{th} origin to the j^{th} destinations in such a way that the total cost is minimized. There can be no shipments from destinations to origins, which is to say $x_{ij} \geqq 0$ for all ij. Symbolically, we may state the problem

letting:

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \qquad \text{(A-1)}$$

where $\quad x_{ij} \geqq 0$ for all i and j

minimize z subject to the restraints

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,\ldots n \qquad \text{(A-2)}$$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,\ldots m$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$x_{ij} \geqq 0 \qquad \text{for all } i \text{ and } j$$

Restating the transportation problem as a standard linear program in the matrix algebra form

$$\text{minimize} \qquad z = C^T X \qquad \text{(A-1')}$$

subject to the restraint

$$\text{where} \qquad AX = B \qquad \text{(A-2')}$$

$$C^T = \left[c_{ij}\right],$$

a transposed vector of c_{ij}'s (all vectors here are assumed to be vertical arrays, hence, any horizontal array is a transpose).

$$X = \left[x_{ij}\right]$$

$$A = \begin{bmatrix} 1100 \ldots 0 \\ 0011 \ldots 0 \\ \vdots \\ 00 \ldots 11 \\ 1010 \ldots 101 \end{bmatrix} \qquad \text{(A-3)}$$

the matrix of coefficients of the system of equations:

$$\sum_j x_{ij} = a_i \qquad i = 1 \ldots m$$

$$\sum_i x_{i1} = b_1$$

we delete the equation for b_2, since any one of the equations may be considered redundant, and

$$B = \left[\begin{array}{c} a_i \\ \hline b_j \end{array}\right]$$

the adjoined vectors of supplies and demands.

The investigations of sensitivity can be stated in the following ways:

I. Variations in C^T

Assuming the already-found minimum basic feasible solution is X_o, since it remains a minimum basic feasible solution if and only if

$$z_j \leq C_j \qquad \text{for all } j$$

where

$$X_o = P^{-1}P_o$$

$$X_j = P^{-1}P_j$$

and P^{-1} is the inverse of P, the $(m + n) \times (m + n)$ matrix composed of column vectors of A.

Then it is true that if

$$(\Delta\bar{C})^T X_j - \Delta C_j \leq z_j - C_j$$

the solution X_o remains minimum, i.e., in investigating the effect of cost on the minimum we first solve (A-1) to see if our old solution is still good. If not, we must resolve.

II. Changes in B

Changes in B, principally in the partition of it involving the amounts to be supplied, are of the most interest in this problem since

there is some probability that individual recruit stations will not meet their quotas.

Since the altered B, B' may be described $B' = B + \Delta B = P'_o$, changes in the minimum basic feasible solution satisfy

$$X'_o = X_o + \Delta X_o$$

$$X'_o = P^{-1} P_o$$

and X'_o will be a basic feasible solution, if and only if

$$x'_j > 0 \qquad \text{for all } x'_j \text{ members of } X'_o.$$

The changed minimum cost (objective function) z'_o is

$$z'_o = \bar{C}^T X'_o$$

$$= z_o + \bar{C}^T \Delta X_o$$

i.e., $$\Delta z_o = \bar{C}^T \Delta X_o \qquad \text{(A-3)}$$

Since the values of B are subject to statistical variation, the planner responsible for budgeting transportation will want to know how such variation will affect his cost, and equation (A-3) gives an indication of how to go about it, as we observe later in Chapter 4, Statistical Variation. For a complete discussion of sensitivity analysis, see Garvin [3].

APPENDIX B

PROBABILISTIC FORMULATION

In this discussion, extensive use will be made of the symbols and ideas presented in Appendix A and some new ones will be introduced. A full list of symbol definitions is presented in the Glossary, Appendix B.

In the problem under consideration we are aided in our probability investigations by the fact that of the system of equations describing the transportation problem, there is always one redundant equation which may be discarded. Since the inputs from the RS's are the members of B of most interest, we discard an equation which describes the input to one of the RTC's. We now assume that the distributions of the RS outputs, $F_{\beta}(b)$, are independent and of the same form although they have different parameters. Of course, the assumption of independence may be attacked, as it implies that the events being assigned the numerical values b, i.e., the number of enlistments at the RS's, do not depend on any common phenomenon such as prosperity, war fever, etc. (We do not include the effect of seasonal fluctuations since, although they probably affect the rate of enlistment over time intervals shorter than a year, they probably have no effect on the annual rate).

A complication of notation is that when we formulate the problem in the simplex notation, we note that

$$B = \left[\beta_j\right] \text{ corresponds to } \begin{bmatrix} a_i \\ \vdots \\ a_m \\ b_j \end{bmatrix} \text{ of the}$$

transportation notation and the b_j of the latter are restricted so that

$$\sum_i a_i = \sum_j b_j$$

or in the notation of our probabilistic simplex

$$\sum_{i=1}^{m} \beta_i = \sum_{j=m+1}^{m+n} \beta_j$$

Fortunately in this problem (as indicated above) $n = 2$ and since the $(m + n)^{th}$ equation is redundant and is discarded, there is only one random variable β_j, namely β_{m+1}, that is a function of the others. The vector B becomes

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_j \\ \vdots \\ \beta_m \\ \\ \beta_{m+1} \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_j \\ \vdots \\ \beta_m \\ \\ \gamma \sum_{j=1}^{m} \beta_j \end{bmatrix} .$$

Since we know the proportion γ, of the total number of recruits $\sum \beta_j$ which the first Recruit Training Center is assigned, we have written

$$\beta_{m+1} = \gamma \sum_{j=1}^{m} \beta_j \quad .$$

In order to obtain the form of the probability distribution function for β_j, $j = 1,\ldots m$ the following reasoning was used:

A recruiter has some probability, p, of enlisting any candidate he interviews. If these probabilities are assumed identical for any one Recruiting Station, the number enlisted in N interviews is a random variable described by a binomial distribution with parameters N and p. However, the number of interviews held is largely out of the

control of the station, that is, it is controlled in only a general way and is not predictable. Energetic RS personnel can raise the local level of interest but must wait for candidates to show that interest. If time periods are properly chosen it may be said that the number of people arriving to be interviewed at an RS is a random variable obeying the following restrictions:

1. The probability of one arrival in a period Δt in length is roughly proportional to the average rate of arrivals and to the length of Δt.
2. The probability of zero arrivals in a period Δt long is roughly one minus the probability for one arrival.
3. The probability of two or more arrivals in a period Δt is of a much smaller order than those in 1 and 2.
4. The arrival of any one person is independent of that of any other.

These are the standard assumptions for the establishing of a Poisson probability distribution law, and the steps used in arriving at it are discussed in elementary probability texts (<u>vide</u> Parzen [5]). We will simply say that N (the number of interviews held in a RS in some convenient time interval Δt) is described by the mass function

$$P[N = n] = e^{-\alpha t} \frac{(\alpha t)^n}{n!} \qquad n = 0,1,2,\ldots$$

$$= 0 \qquad \text{otherwise}$$

where α is the average number of arrivals per period (e.g., per recruiting day).

Now the number of men actually enlisted in N interviews is seen to be a random variable described by a conditional distribution function.

For the j^{th} station

$$F_{\beta_j|N_j}(b_j|n_j) = P\left[\beta_j \leq b_j \middle| N = n\right]$$

There is one more complicating detail, which is that the recruiter is not able to stop recruiting until he has filled his quota, q_j. In effect, this truncates the binomial distribution governing the "output" of his n_j interviews, so that

$$0 \leq \beta_j \leq n_j \qquad \text{if } n_j \leq q_j$$

and

$$0 \leq \beta_j \leq q_j \qquad \text{if } q_j < n_j$$

Dropping the j subscripts for clarity, then

$$F_{B|N}(q|n) = \sum_{\{b|p(b|n)>0\}} p(b|n)$$

$$= K_1 \sum_{i=0}^{n} \binom{n}{i} p^i r^{n-i} = 1 \qquad \text{if } n \leq q$$

$$= K_2 \sum_{i=0}^{q} \binom{n}{i} p^i r^{n-i} = 1 \qquad \text{if } q < n$$

so that $K_1 = 1$

and $$K_2 = \frac{1}{\sum_{i=0}^{q} \binom{n}{i} p^i r^{n-i}}$$

where $r = (1 - p)$

now $$F_{\beta}(b) = \sum_{\{n|p(n)>0\}} F_{\beta|N}(b|n)\, p_N(n)$$

$$= \sum_{n=0}^{q} (\alpha t)^n \frac{e^{-\alpha t}}{n!} \sum_{i=0}^{b} \binom{n}{i} p^i r^{n-i}$$

$$+ \sum_{n=q+1}^{\infty} (\alpha t)^n \frac{e^{-\alpha t}}{n!} \frac{\sum_{i=0}^{b} \binom{n}{i} p^i r^{n-i}}{\sum_{i=0}^{q} \binom{n}{i} p^i r^{n-i}}$$

$$= \sum_{i=0}^{b} \left[\sum_{n=0}^{q} (\alpha t)^n \frac{e^{-\alpha t}}{n!} \binom{n}{i} p^i r^{n-i} + \frac{\sum_{n=q+1}^{\infty} (\alpha t)^n e^{-\alpha t} \binom{n}{i} p^i r^{n-i}}{\sum_{k=0}^{q} \binom{n}{k} p^k r^{n-k}} \right]$$

The size and complexity of this expression suggests the desirability of resorting to a computer when computations must be carried out, and for some of our purposes we can do so by use of Monte Carlo simulation techniques.

The planner responsible for budgeting the transportation of recruits will wish to know not only the "best" way to transport them, but also what the effort is going to cost. A common measure of cost used in such situations as this, where there is uncertainty involved, is to determine the expected value of the cost; in this case it would be of minimum cost. Ideally the entire probability distribution of minimum cost is desired, but problems of this nature are often found to be too difficult to solve entirely. In this case even if we could solve for the expected value of minimum cost, we would not have arrived at

a very satisfactory answer to the question, "How cheaply can this transportation be carried out?" The reason the answer is unsatisfactory is that implicit in the use of the expected value of minimum cost to answer the question, is the assumption that a real system could be set up to handle any transportation input optimally. Instead of taking such an ideallistic approach we have decided to be satisfied with trying to find a solution which will deviate least from this ideal. The "analytical" expression for the probability distribution of recruiting station inputs to the transportation problem (derived above) gives some idea of the complexity of an analytical approach to finding this expected value. In order to avoid the difficulty, the FORTRAN program for solving transportation problems was modified to solve a large number of ramdomized problems when given inputs of quotas, probability of interview success, and average rate of arrivals for each RS. Thus, it was possible to generate a vector of random inputs and to obtain a sample mean value of minimum costs. In order to give an indication of how the distribution of minimum costs varies the program was designed to solve for sample deviation also. Here again it should be noted that since many data were picked at random (i.e., the values of p_j, the probability of interview success, and α_j the average arrival rate for the j^{th} RS), the solution is only an illustration of method.

From the previously mentioned relation that

$$E\left[z_o(B)\right] \leq z_o\,(E[B]) \qquad (4.1)$$

we see that a solution of the transportation problem using mean values of inputs will give us an upper bound on the set of possible means of optimal solutions. It is also one specific description of the input

situation for which we can find a minimum cost transportation pattern, but it may not be the "best".

What we seek for the "best" is first, a loss function describing the penalty we pay for improper action, and second, a rule for deciding what action to take so that we can expect to pay the least penalty. In other words, we are trying to solve a standard statistical decision problem and are trying to determine an estimator. Desirable properties of such estimators are discussed in [6] and other standard statistics texts. Those properties sometimes conflict and are often chosen somewhat arbitrarily. Our estimator will be chosen arbitrarily also.

Where we first define

$$\hat{z} = \min z(\hat{B})$$

for $\hat{B}$, an estimator of B, a natural loss function,.., would seem to be

$$\lambda(\hat{B};B) = \hat{z}(B;\hat{B}) - z_o(B)$$

where we write $\hat{z}(B;\hat{B})$ to emphasize the parametric role played by $\hat{B}$. Later it will also be written without the $\hat{B}$.

The risk, or expected loss, we wish to minimize is

$$R(\hat{B};B) = E\left[\lambda(\hat{B};B)\right]$$

where $\hat{B}$ represents the decision rule for arriving at the final action, the appropriate transformation on B.

Our rule, "use the estimator $\hat{B}$", is that which consists of acting as though B were the actual value of B. The action then is

$$\hat{z} = C^T \hat{X}$$

$$\hat{z} = C^T \hat{P}^{-1} B$$

where we now regard $\hat{z}$ as a transformation on $\hat{B}$. The vector $\hat{X}$, the "minimum basic feasible solution" is one of the results of the solution of our transportation problem under the restrictions represented by $\hat{B}$. Equation (4.1) becomes

$$\min R = \min_{\hat{B}} E\left[\hat{z}(B) - z_o\,(B)\right]$$

In words, equation (4.1) regards the loss as the difference between a fixed solution and an ideal solution. The fixed solution is that arrived at by use of the estimator $\hat{B}$. The ideal solution is that arrived at by use of the actual value of B.

Drawing on the notation of simplex algorithm one is tempted to say

$$\min R = \min_{\hat{B}} \left\{C^T\hat{P}^{-1}(E[B]) - E\left[C^T\hat{P}^{-1}B\right]\right\}$$

making use of the linearity of the transformation represented by $\hat{P}^{-1}$, that transformation arrived at by a simplex solution using the estimator $\hat{B}$. In fact, the authors took this approach until certain computer outputs brought the inherent error (discussed below) to their attention. Continuing the above reasoning would have led to the following satisfying solution.

Letting $E[B] = \bar{B}$, and substituting, equation (a) becomes

$$E\left[C^T P^{-1} B\right] \leqq C^T \bar{P}^{-1} \bar{B}$$

where $\bar{P}^{-1}$ is the transformation arrived at by use of $E[B]$ in the simplex

algorithm, and by the definition of

$$z_o\,(B) = \min\left\{z(B)\right\} = \min_{P^{-1}}\left\{C^T P^{-1} B\right\},$$

$$C^T\bar{P}^{-1}\bar{B} \leq C^T P^{-1}\bar{B}$$

for any other $P^{-1} \neq \bar{P}^{-1}$, therefore,

$$\min R = \min_{\hat{B}}\left\{C^T\hat{P}^{-1}\bar{B} - E\left[C^T P^{-1} B\right]\right\}$$

$$\min R = \left\{C^T\bar{P}^{-1}\bar{B} - E\left[C^T P^{-1} B\right]\right\}$$

i.e.,

$$\hat{B} = \bar{B}$$

and

$$\hat{P}^{-1} = \bar{P}^{-1},$$

the best policy is to use the mean value of B as an estimator and the transportation pattern computed from it.

The above approach is in error in that we assumed we could use the transformation $\hat{P}^{-1}$ over the entire space of B vectors with some of the solutions becoming non-optimal, ignoring the possibility that they might become infeasible. However, the fact is that the solution to the linear programming problem is either optimal or else it is infeasible, where the term feasibility is technically defined. For a solution to be feasible it must satisfy the restrictions

$$AX = B$$

and $\quad x_i \geq 0 \quad$ for all i. $\quad$ (4.2)

For the mathematical details of this aspect of the simplex algorithm, the reader is referred to Garvin [3].

We can restate the definition of feasibility by defining the elements of the P^{-1} matrix as p_{ik}^{-1} and writing for equation (4.2) (in the case where we started with $\hat{B}$ and its implicit transformation $\hat{P}^{-1}$)

$$\sum_{k=1}^{m} \hat{p}_{ik}^{-1} \beta_k < 0 \quad ,$$

and we want to know the probability of this occurring, i.e.,

$$P\left[\sum_{k=1}^{m} p_{ik}^{-1} \beta_k < 0\right]$$

The implication of the fact that $\hat{P}^{-1}$ B is optimal or else not feasible is that there are two regions in the space of B vectors and they are disjoint. In one region the loss function is zero-valued and $\hat{P}^{-1}$ is optimal. In the other, the loss function takes on a positive value, so apparently the estimator $\hat{B}$ that we seek is that one which is associated with the region of infeasibility with the least expectation. That is, the risk can be more accurately defined as

$$R(\hat{B};B) = 0 + \int_{\{B \; \hat{P}^{-1}B \notin K\}} \lambda(\hat{B};B)dF_B$$

where K is the set of all feasible solutions to the equation AX = B. Thus, the estimator we seek is that which minimizes the second term on the right, i.e.,

$$\min R(\hat{B};B) = \min_{\hat{B}} \left\{ \int_{\{B | \hat{P}^{-1}B \notin K\}} \lambda(\hat{B};B)dF_B \right\}$$

Now, defining $X^* = \hat{P}^{-1}B$, and x^* as some vector value,

$$\lambda(\hat{B};B) = 0 \qquad \text{if } X^* \varepsilon\ K$$

$$> 0 \qquad \text{if } X^* \not\varepsilon\ K, \text{ i.e., } X^* \varepsilon\ \bar{K}$$

$$P[X^* \varepsilon\ K] = \int_K dF_{X^*}(x^*)$$

$$P[X^* \not\varepsilon\ K] = \int_{\bar{K}} dF_{X^*}(x^*)$$

$$= \int_{\bar{K}} f_{X^*}(x^*)dx^*$$

where $\bar{K}$ is the compliment of K in the space of all $\hat{P}^{-1}B$,

and

$$f_{X^*}(x^*) = f_{X_1, X_2 \ldots X_m}(x_1 \ldots x_m)$$

$$= f_{\beta_1, \beta_2, \ldots \beta_m}(b_1, b_2 \ldots b_m) \left| J(b_1, \ldots b_m) \right|^{-1}$$

Now

$$J(b_1, \ldots b_m) = \begin{vmatrix} \frac{\partial x_1}{\partial b_1} & \cdots & \frac{\partial x_1}{\partial b_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_m}{\partial b_1} & \cdots & \frac{\partial x_m}{\partial b_m} \end{vmatrix} = \left| P^{-1} \right| \neq 0$$

therefore

$$f_{X^*}(x^*) = f_{\beta_1, \ldots \beta_m}\left(\sum_{i=1}^{m} p_{1i}x_i, \ldots \sum_{i=1} p_{mi}x_i \right) \left| P \right|$$

Here a difficulty arises due to the non-independence of β_{m+1} and β_i, $i = 1, \ldots m$.

However, assuming the foregoing operations have been carried out, we can make use of the central limit theorem to say that the random variables β_i, i = 1,...m, can be said to be approximately normally distributed since they are large sums of random variables, at least over long periods of time. Further, since β_{m+1} is also a large sum of these sums, it can be approximated also as a normally distributed random variable which is independent of the other m. Consequently, we are able to write for these approximations

$$f_B(b) = f_{\beta_1,\ldots\beta_m}(b_1,\ldots b_m)$$

$$\cong \prod_{i=1}^{m} f_{\beta_i}(b_i)$$

i.e., there is a density function which can serve our purpose.

Now

$$E\left[\lambda(\hat{B};B)\right] = \int_{\{X^* | X^* \varepsilon K\}} \lambda(\hat{B}(\hat{X});B(X^*))dF_{X^*} .$$

To carry this out we must redefine $\hat{z}$ to conform to reality, e.g.,

$$\hat{z}(B) = \sum_{i=1}^{m} c_i x_i^{*'}$$

$$\text{where} \quad x_i^{*'} = \begin{cases} 0 & \text{if } x_i^* < 0 \\ x_i^* + x^* & \text{if } x_{i-1}^* < 0 \text{ and } i \text{ is odd} \\ x_i^* + x_{i+1}^* & \text{if } x_{i+1}^* < 0 \text{ and } i \text{ is even} \\ x_i^* & \text{otherwise} \end{cases}$$

(This definition of $x_i^{*'}$ models the action of an RS in carrying out a

fixed policy). Finally

$$E[\lambda]=\int_{-\infty}^{0}\int_{-\infty}^{0}\cdots\int_{-\infty}^{0}\left(\sum_{i=1}^{m}c_i x_i^{*'}-\sum_{i=1}^{m}c_i x_{i0}\right)f_{\beta_1,\ldots\beta_m}\left(\sum_{i=1}^{m}p_{1i}x_i,\ldots\sum_{i=1}^{m}p_{mi}x_i\right)\left|P\right| dx_1,\ldots dx_m$$

Here again we have come to a point where for computational purposes, theory yields to simulation and the theory merely provides a conceptual framework for what we are trying to do. Of the ideas written above, at least the loss function can be useful in the simulation.

In investigating the results of the above reasoning by Monte Carlo simulation, a program was designed to provide the randomized input B vectors and solve for their sample mean (one common estimator). This mean vector was then used as an input. The program computed the results of transforming the randomized B vectors (using the transformation found in the solution for the mean value vector) altering it by using the definition given above for $x_i^{*'}$ outside the region of basic feasibility. It further computed the loss function in each case and the sample mean loss or sample risk for this estimator. In computing the random B vectors, the same concept of the events giving rise to enlistments was used as in the analytical description; however, because of the large numbers involved, and to speed computation time, a normal approximation to the Poisson distribution was used. For this approximation see Parzen [5], and for the formula used to compute random numbers according to a normal distribution see Vaa [9].

We are able to find sample risks given <u>specific</u> B estimators but we do not believe it is possible to solve for the best. The only one for which we have solved so far is the mean B vector, $\bar{B}$. For examples

of the results of this simulation (using completely arbitrary parameters) see Tables VII-A through VII-G, pages 55 to 61, and for a discussion of the results see Section 6.

TABLE VII-A

PROBLEM RECRUIT *SAMPLE*

NUMBER OF SAMPLES 15

ROUTE NUMBERS AND ASSOCIATED CCSTS

1	52.23	2	161.92	3	34.27	4	154.46	5	23.13	6	119.55
7	37.23	8	147.51	9	16.70	10	138.40	11	28.06	12	133.42
13	11.79	14	129.95	15	29.93	16	142.34	17	31.31	18	130.74
19	22.60	20	118.44	21	43.99	22	128.56	23	38.63	24	130.35
25	30.97	26	128.44	27	15.50	28	118.18	29	47.09	30	131.87
31	11.55	32	106.56	33	17.25	34	126.16	35	12.73	36	117.29
37	31.21	38	142.01	39	22.54	40	124.72	41	1.28	42	97.17
43	14.65	44	122.92	45	11.70	46	109.08	47	2.23	48	107.10
49	8.81	50	98.49	51	27.53	52	57.96	53	15.04	54	99.05
55	18.72	56	79.71	57	13.92	58	95.70	59	19.00	60	86.19
61	42.72	62	31.31	63	33.25	64	47.18	65	66.41	66	76.91
67	20.37	68	91.55	69	35.14	70	103.02	71	29.52	72	57.35
73	97.81	74	3.36	75	78.41	76	31.50	77	91.35	78	14.88
79	106.29	80	23.36								

TABLE VII-B

NUMBER OF RECRUITING DAYS IN PERIOD 365

GREAT LAKES PERCENT .500

STATION NUMBER	QUOTA	AVERAGE INTERVIEW NUMBER	PROB OF INTERVIEW SUCCESS
1	2560.	10.	.550
2	4460.	18.	.650
3	2147.	9.	.500
4	7351.	42.	.700
5	1361.	6.	.650
6	1399.	6.	.500
7	1399.	6.	.650
8	1399.	6.	.700
9	830.	3.	.750
10	2054.	9.	.600
11	1478.	6.	.750
12	2445.	12.	.700
13	1358.	6.	.750
14	2054.	9.	.750
15	1478.	6.	.700
16	1674.	6.	.750
17	2929.	12.	.750
18	1395.	6.	.650
19	4882.	18.	.800
20	3070.	9.	.600
21	4563.	18.	.500
22	4333.	15.	.600
23	1663.	8.	.550
24	1922.	8.	.600
25	2777.	12.	.550
26	1808.	8.	.650
27	1864.	9.	.600
28	2504.	9.	.700
29	3380.	15.	.750
30	1673.	8.	.700
31	1432.	6.	.700
32	1887.	9.	.750
33	2539.	12.	.700
34	1363.	6.	.700
35	1244.	6.	.450
36	1412.	6.	.600
37	6201.	27.	.700
38	2146.	9.	.650
39	4801.	21.	.700
40	2750.	12.	.750
TOTAL	99990.		

TABLE VII-C

SAMPLE 12

STATION NUMEER	NUMBER MEN ENLISTED	PERCENT CUCTA FILLED
1	1988.	77.656
2	4196.	94.081
3	1643.	76.525
4	7351.	100.000
5	1361.	100.000
6	1094.	78.199
7	1399.	100.000
8	1399.	100.000
9	819.	98.675
10	1989.	96.835
11	1478.	100.000
12	2445.	100.000
13	1358.	100.000
14	2054.	100.000
15	1478.	100.000
16	1624.	97.013
17	2929.	100.000
18	1357.	97.276
19	4882.	100.000
20	1973.	64.267
21	3330.	72.978
22	3112.	71.738
23	1603.	96.392
24	1740.	90.531
25	2384.	85.848
26	1808.	100.000
27	1864.	100.000
28	2338.	93.371
29	3380.	100.000
30	1673.	100.000
31	1432.	100.000
32	1887.	100.000
33	2539.	100.000
34	1363.	100.000
35	977.	78.537
36	1314.	93.059
37	6201.	100.000
38	2146.	100.000
39	4801.	100.000
40	2750.	100.000
41	46729.	93.467

TABLE VII-D

MINIMUM COST OF ALL TRANSPORTATION IS 4142953.99

ROUTES USED AND AMOUNTS

ROUTES	NUMBER MEN SHIPPED
1	1988.
3	4196.
5	1643.
7	7351.
9	1361.
11	1094.
13	1399.
15	1399.
17	819.
19	1141.
47	1740.
41	3330.
25	1358.
27	2054.
45	1603.
32	1624.
33	2929.
35	1357.
37	4882.
39	1973.
20	848.
22	1478.
24	2445.
30	1478.
50	2384.
52	1808.
54	1864.
56	2338.
58	3380.
60	1673.
62	1432.
64	1887.
66	2539.
68	1363.
70	977.
72	1314.
74	6201.
76	2146.
78	4801.
80	2750.
43	3112.

AVERAGE COST	STD DEVIATION OF COST
4159224.85	11172.45

TABLE VII-E

STATION NUMBER	MEAN NUMBER ENLISTED
1	2049.
2	4262.
3	1579.
4	7351.
5	1361.
6	1051.
7	1399.
8	1399.
9	806.
10	1998.
11	1478.
12	2445.
13	1358.
14	2054.
15	1478.
16	1606.
17	2929.
18	1395.
19	4882.
20	1985.
21	3180.
22	3344.
23	1576.
24	1779.
25	2409.
26	1808.
27	1864.
28	2255.
29	3380.
30	1673.
31	1432.
32	1887.
33	2539.
34	1363.
35	953.
36	1312.
37	6201.
38	2050.
39	4801.
40	2750.
41	46710.

TABLE VII-F

MINIMUM COST OF ALL TRANSPORTATION OF MEANS IS 4154715.82

ROUTES USED AND AMOUNTS

ROUTES	NUMBER MEN SHIPPED
1	2049.
3	4262.
5	1579.
7	7351.
9	1361.
11	1051.
13	1399.
15	1399.
17	806.
19	971.
22	1478.
41	3180.
25	1358.
27	2054.
47	1779.
32	1606.
33	2929.
35	1395.
37	4882.
39	1985.
20	1027.
30	1478.
45	1576.
24	2445.
50	2409.
52	1808.
54	1864.
56	2255.
58	3380.
60	1673.
62	1432.
64	1887.
66	2539.
68	1363.
70	953.
72	1312.
74	6201.
76	2050.
78	4801.
80	2750.
43	3344.

TABLE VII-G

SAMPLE	COST USING RANDOM INPUT	LOSS ON THIS SAMPLE
1	4163734.78	.00
2	4167899.97	.00
3	4154345.61	.00
4	4167284.43	.00
5	4163171.33	.00
6	4170892.39	.00
7	4159545.75	.00
8	4168555.26	.00
9	4168189.51	.00
10	4144859.98	.00
11	4139265.22	.00
12	4142953.99	.00
13	4164795.48	.00
14	4158533.09	.00
15	4154715.82	.00

RISK .00 STD DEVIATION OF LOSS .00

APPENDIX C

OTHER POSSIBLE LINES OF INVESTIGATION

1. When a planner decides to seek a solution to a problem by using the criterion of minimum expected loss, he is considering all events (in our case B vector values) which have any positive probability of occurring. Suppose, however, he should prefer to make a wager based on the assumption that in the finite period of time his system (the U. S. Navy) is in operation, that events of extremely small probability will not occur. By basing his actions on such an assumption he is betting against nature that he can save an amount of money in this finite period of time, realizing that he risks losing more than the amount he saves, but risks it with a very low probability that the (extra) loss will occur. In effect, in our case he wishes to find the solution (B estimator) for the following objective function:

$$\min_{\hat{B}} \left\{ \int_{\lambda(B;\hat{B})_\theta}^{\lambda(Q;\hat{B})} \left[\hat{z}\ (B;\hat{B}) - z_o(B) \right] dF_L \right\}$$

where

θ represents the upper limit of the small level of probability of the events he wishes to bet will not occur,

$\hat{z}\ (B;\hat{B})$ is the objective function solution of the transportation problem for $\hat{B}$ evaluated at B and emphasizes the parametric role played by $\hat{B}$.

dF_L is the differential of the probability distribution function for $\lambda(B;\hat{B})$. $\lambda(B;\hat{B})_\theta$ is the θ^{th} percentile of $\lambda(B;\hat{B})$ under the probability law F_L.

That the bet mentioned above might be a desirable one to make

could be determined in the following way. Suppose, as is likely, that the expected values of the β's, the components of our random vector B, occur near their maximum values, the q's. The reason this is likely is that the quotas of recruiting stations would probably be assigned in just such a manner. Also suppose $\bar{B}$ is our best estimator using expectation. Further, suppose a sensitivity analysis showed that small changes of $\bar{B}$ with values of β increasing should cause the solution to become non-optimal, i.e., the loss for small increments of β increases rapidly and should not satisfy the constraint that $x_i \geq 0$ for all x_i of the solution; and that equal small changes of $\hat{B}$ with values of β decreasing should not cause the solution to become non-optimal, i.e., for small decrements of β, the loss remains at zero. The probability mass function of one of such β's, $p_\beta(b)$ would have a form similar to Figure A.

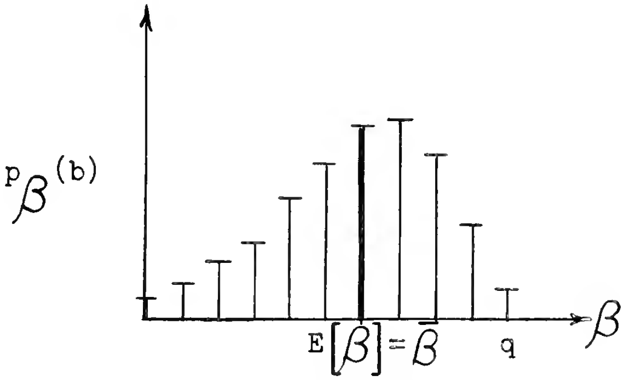

FIGURE A

From the figure we see that the probability that β takes on the exact value $(\bar{\beta} + \Delta\beta)$ is greater than the probability that it takes on the exact value $(\bar{\beta} - \Delta\beta)$

$$p_\beta\ (\bar{\beta} + \Delta\beta) > p_\beta\ (\bar{\beta} - \Delta\beta)$$

where $\Delta\beta$ is some arbitrary small change in β. This statement hinges on our assumption of the level of $\bar{\beta}$ relative to q and the fact that q is large, i.e., that $\bar{\beta}$ is on the increasing side of the mode of $p_{\beta}(b)$ and so are both $(\bar{\beta} + \Delta\beta)$ and $(\bar{\beta} - \Delta\beta)$.

Looking in the same general way at the loss function, we see that it will probably have a form similar to that of Figure B,

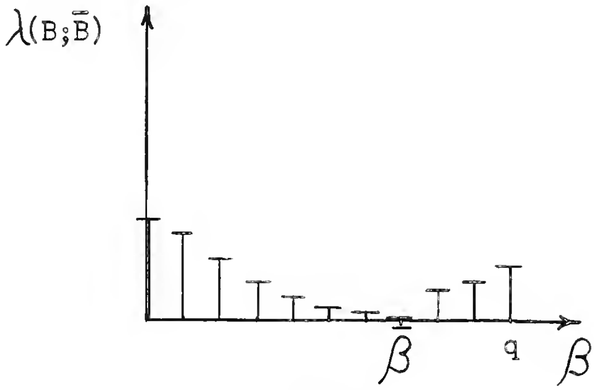

FIGURE B

since there will obviously be no loss at $\bar{\beta}$. It might (and a sensitivity analysis would determine this) look like Figure C,

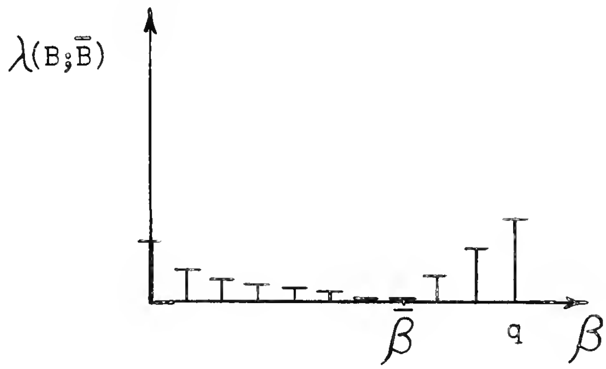

FIGURE C

so that, in the latter case the probability density function of $\lambda(B;\hat{B})$ might be similar to Figure D.

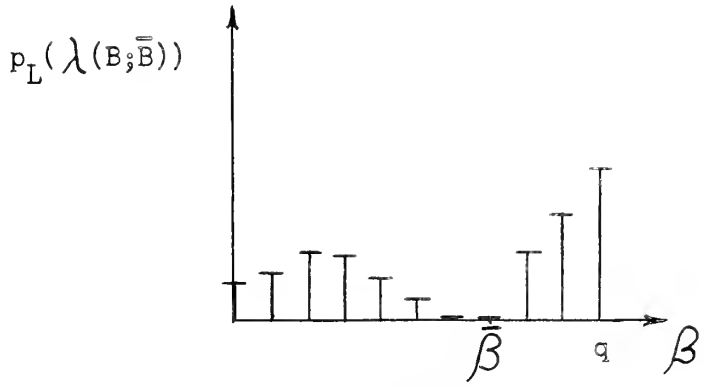

FIGURE D

Even so simple a discussion as that above makes it clear that, should these conditions hold, for all (or possibly even some smaller number) of the components of B, an estimator of B with components to the right of the prospective $\bar{\beta}$'s would be a reasonable choice, given the willingness to make the previously mentioned bet against nature. The choice of such a new estimator, $\widetilde{B}$, might result in a loss function such as that in Figure E,

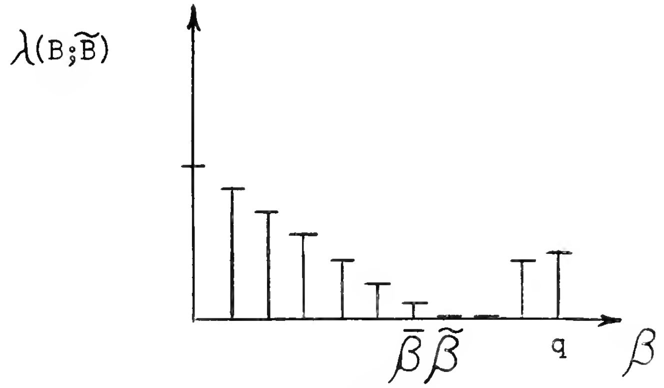

FIGURE E

and a probability mass function similar to that of Figure F.

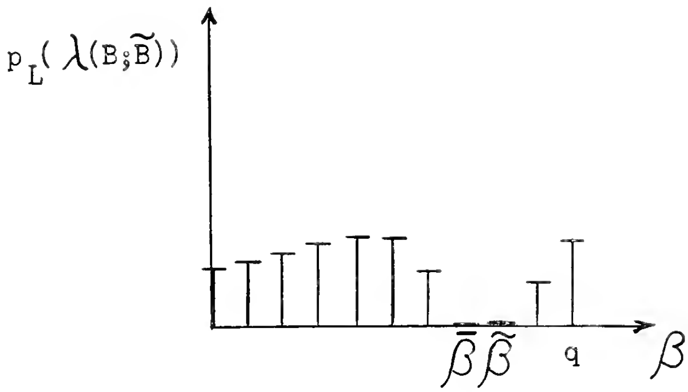

FIGURE F

The small standard deviations resulting from the parameters we used in our simulations, indicate that this second approach would not be profitable. However, were the numbers of recruits sufficiently small to "spread out" the probability mass functions more, an investigation of this additional approach might become profitable.

The effect of ignoring the tail, that is, those losses with probability less than Θ, is illustrated in Figure G. In effect the new $\hat{B}$, $\tilde{B}$, can be found by renormalizing F_L truncated at the Θ^{th} percentile and finding the new mean of the region of B vector which produce the truncated function. An approximation to this new mean (a new sample mean) could easily be found by use of a computer.

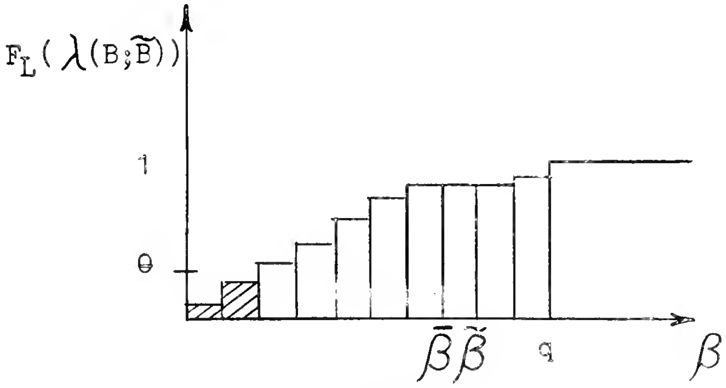

FIGURE G

2. A possible improvement that might be made in the solution of this problem of the transportation probability model would be obtained by investigating the problem's dynamic aspect. A more sophisticated model than ours might allow the planner to make an optimal decision at any point in the time period (one, n, or an "infinite" number of years) based upon the conditions then prevailing. It is believed that a multi-staged linear program such as is discussed by Dantzig [4] and [7], or a dynamic program approach such as those discussed by Bellman [8] could be used to provide this dynamic aspect.

APPENDIX D

GLOSSARY OF SYMBOLS

A	The matrix of coefficients of the system of linear equations represented by equations (A-2) and (A-2'). The matrix is defined in (A-3).
a_i	The total number of recruits to be shipped from the i^{th} RS, of which there are m.
α_j	The average number of candidates arriving per day at the j^{th} RS.
B	The adjoint column vector composed of the a_i's and b_j's.
$\hat{B}$	A fixed value estimator of B.
$\bar{B}$	The vector of expected values of the random variables β_j.
$\breve{B}$	A (second) fixed value estimator of B.
β_j	The components of the vector B when the vector is considered as a vector of random variables.
b_j	A fixed value in the domain of β_j. Also the total number of recruits to be shipped to the j^{th} RTC, of which there are n.
C^T	The transposed column vector of c_{ij}'s.
c_{ij}	The total cost of shipping one recruit from the i^{th} RS to the j^{th} RTC.
γ	The proportion of total input assigned to the RTC of interest.
$E[V]$	The expected value of some random variable V in which V might be a scalar or vector. In the latter case, the symbol represents a vector of expected values.
F_v	The probability distribution function of some random variable V.
$J(\ldots)$	The Jacobian determinant of a transformation with respect to the variables $(\ldots)$.
K	The set of all feasible solutions to the linear program for $AX = \hat{B}$.
$\bar{K}$	The complement of K in the space of all possible $\hat{P}^{-1} B$ vectors.

$\lambda(B;\hat{B})$	A loss function, the difference between the least cost possible if the problem could be solved optimally for B, and the cost obtained using the transformation derived by using $\hat{B}$ as the estimator.
N_j	The number of interviews occurring at the j^{th} RS.
P^{-1}	The inverse of the matrix composed of a set of linearly independent vectors, used as a transformation on B to yield X.
$\hat{P}^{-1}$, $\bar{\bar{P}}^{-1}$	Those transformations arrived at by use of $\hat{B}$ and $\bar{\bar{B}}$ respectively in simplex solutions of the transportation problem.
$\hat{p}^{-1}_{ik}$	The i,k^{th} element of matrix $\hat{P}^{-1}$.
$p_V(v)$	The probability mass function of some random variable V.
p_j	The probability that the j^{th} recruiter will enlist a candidate at an interview.
Q	The vector of q_j's.
q_j	The assigned quota of recruits for the j^{th} RS.
$R(\hat{B};B)$	A regret function or expected value of loss.
r_j	$r_j = 1 - p_j$
θ	The level of probability of the events which the planner bets will not occur.
X	The column vector composed of the x_{ij}'s.
x_{ij}	The total number of recruits shipped from the i^{th} RS to the j^{th} RTC.
X_o	The vector of x_{ij}'s which yields the minimum z for the general case.
x_{io}	An element of X_o.
$\hat{X}$	The vector of x_{ij}'s which yields the minimum z when $\hat{B}$ is the vector of restrictions.
X^*	The vector valued random variable resulting from the operation of the transformation $\hat{P}^{-1}$ on the random vector B.
x^*	A possible value of X^*.
X^*_i	The i^{th} component of X^*.
x^*_i	The i^{th} component of x^*.

$x_i^{*'}$ The transformed x_i^* which models the actual situation of an RS carrying out a policy.

z The objective function expressed in equations (A-1) and (A-1').

z_o The transformation which yields the minimum objective function, i.e., $C^T X$.

$z_o(B)$ The minimizing transformation found using B as a set of parameters and considering B as a vector of random variables.

$\hat{z}\ (\hat{B})$ The minimizing transformation found by using $\hat{B}$ as the specific value of B.

$\hat{z}\ (B)$ The transformation of B which gives the minimum of the objective function when acting on $\hat{B}$.

APPENDIX E

```
      PROGRAM LP2                                                       LP200010
C                                                                       LP200011
C     A MODIFICATION OF LP1 TO GENERATE A TRANSPORTATION MATRIX         LP200012
C                                                                       LP200011
      DIMENSION A(50,100),B(50),KB(100),X(50),JH(50),P(50)              LP100020
     1     ,Y(50),E(50,50),Z(100),DDT(100),PINV(50,50),XI(50)           LP100010
      DIMENSION INFIX(10),TOL(10),KOUT(10),ERR(10),RUN(8)               LP100050
      EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MROW),(NRUN,RUN(1))         LP100050
      ICASE=1                                                               COA1
    2 GO TO (992,993),ICASE                                                 COA2
  992 DO 10 I=1,100                                                         COA3
      DO 10 J=1,50                                                      LP100070
   10 A(I,J)=0.                                                         LP100080
C                                                                       LP100090
C     PERMANENT DATA                                                    LP100100
C        INFIX (3)=MAX NO. OF ROWS (CF. DIMENSION STATEMENT)            LP100100
C        INFIX(7)=MAX INTERATION COUNT                                  LP100120
C                                                                       LP100130
      B(1)=0.                                                           LP100140
      INFIX(1)=4                                                        LP100150
      INFIX(3)=50                                                       LP100160
      INFIX(5)=2                                                        LP100170
      INFIX(6)=1                                                        LP100180
      INFIX(7)=100                                                      LP100190
      INFIX(8)=0                                                        LP100200
      TOL(1)=1.E-7                                                      LP100210
       TOL(2) = 1.E-5                                                   LP100220
      TOL(3) = -1.E-6                                                   LP100230
      TOL(4) = 1.E-7                                                    LP100240
       PRM = 0.                                                         LP100250
C                                                                       LP100260
C     INPUT --  NOTE.. MROW = NO. OF ROWS PLUS ONE, BECAUSE COST        LP100270
C                     COEFFICIENTS ARE ENTERED AS ROW ONE.              LP100280
C                                                                       LP100290
      ICASE=2                                                               COA4
      READ INPUT TAPE 2,3,NCLL,MRWW,RUN                                 LP200300
    3 FORMAT (2I5,7A8,A6)                                               LP100310
      IF (NRUN-4HSTOP)9999,9998,9999                                    LP200301
 9998 WRITE OUTPUT TAPE 3,9997                                          LP200302
 9997 FORMAT (1H1,10X,12HJOB COMPLETE)                                  LP200303
      STOP                                                              LP200304
 9999 NCOL=NCLL*MRWW                                                    LP200305
      IF (NCOL) 500,500,8                                               LP100320
    8 MROW=NCLL+MRWW                                                    LP200325
      MRWW1=MRWW+1                                                      LP200330
      MRWW2=MRWW+2                                                      LP200335
    5 READ INPUT TAPE 2,4,(A(1,J),J=1,NCOL)                             LP200340
C                                                                       LP200345
```

```
C                   NOTE-  THE A(1,J) ROW ARE THE COST COEFFICIENTS            LP100350
C        THE C-I,J-S ARE  READ IN ROW-WISE                                    LP200355
C                                                                             LP200356
    4 FORMAT (7F10.5)                                                         LP100360
      J=0                                                                     LP200361
      DO 4000  I=2,MRWW1                                                      LP200362
      DO 4000  K=1,NCLL                                                       LP200363
      J=J+1                                                                   LP200364
 4000 A(I,J)=1.0                                                              LP200365
      J=0                                                                     LP200366
      DO 4001  I=MRWW2,MROW                                                   LP200367
      J=J+1                                                                   LP200368
      DO 4001  K=J,NCOL,NCLL                                                  LP200369
 4001 A(I,K)=1.0                                                              LP200370
      MROW2=MROW+2                                                                COA5
      READ INPUT TAPE 2,4,(B(I),I=2,MROW2)                                    LP200372
      B(MROW)=B(MROW)-B(MROW2)                                                    COA6
  993 B(MROW)=B(MROW)+B(MROW2)                                                    COA7
      IF(B(MROW)-B(MROW+1))994,994,992                                            COA8
  994 CONTINUE                                                                   COA10
C                                                                             LP200373
C ******  NOTE-THE ORIGIN INFORMATION IS READ FIRST, FOLLOWED BY THE          LP200374
C              DESTINATION INFORMATION. THE LAST DESTINATION IS EXCLUDED.     LP200375
C                                                                             LP200376
C     WRITE OUTPUT TAPE 3,29, RUN                                             LP100380
   29 FORMAT (1H1,28X,7A8,A6)                                                 LP100390
C     WRITE OUTPUT TAPE 3,100, (A(1,J)J=1,NCOL)                               LP100500
  100 FORMAT (///10X,17HCOST COEFFICIENTS//(10X,8E13.6))                      LP100510
C     DO 105 I=2,MROW2                                                        LP100520
C     K=I-1                                                                   LP100530
C     WRITE OUTPUT TAPE 3,102,K,(A(I,J),J=1,NCOL)                             LP100540
  102 FORMAT (//3X,4HROW I2,1X,8E13.6/(10X,8E13.6))                           LP100560
C 105 WRITE OUTPUT TAPE 3,28,K,B(I)                                           LP100570
   28 FORMAT (5H   B(I2(1H),2X,E13.6)                                         LP100580
C                                                                             LP100590
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E,DDT)               LP100600
C                                                                             LP100610
      WRITE OUTPUT TAPE 3,29,RUN                                              LP100620
      WRITE OUTPUT TAPE 3,30,Y(1),B(MROW)                                     LP100630
   30 FORMAT (///34X38HMINIMUM COST OF OBJECTIVE FUNCTION IS E13.6////        LP100640
     145X,20HGREAT LAKES PERCENT E13.3////                                    LP   641
     145X,30HBASIS VECTORS AND COEFFICIENTS///37X,6HVECTOR,10X,30HCOEFFILP100650
     2CIENT (X-ZERO COMPONENT)//)                                             LP100650
C                                                                             LP100660
C     FOLLOWING IS SET UP FOR ADDITIONAL OUTPUT IF REQUIRED                   LP100670
C                                                                             LP100680
      DO 33 I=1,NCOL                                                          LP100990
      II=KB(I)                                                                LP100700
```

```
      IF(II) 31,31,32                                                   LP100710
   31 Z(I)=0.                                                           LP100720
      GOTO33                                                            LP100730
   32 Z(I)=X(II)                                                        LP100740
   33 CONTINUE                                                          LP100750
      NDE=7                                                             LP100760
      IF (NDE-NCOL) 35,36,36                                            LP100770
   35 NDE=NCOL                                                          LP100780
   36 IF (NDE-MROW)37,40,40                                             LP100790
   37 NDE=MROW                                                          LP100800
C                                                                       LP100810
C     BACK TO NORMAL                                                    LP100820
C                                                                       LP100830
   40 WRITE OUTPUT TAPE 3,43,(JH(I),X(I),I=2,MROW)                      LP100840
   43 FORMAT (37X,2HP(I2,1H),19X,E13.6)                                 LP100850
C                                                                       LP100860
C     WRITE OUTPUT TAPE 3,7733                                          LP100870
 7733 FORMAT(1H1,47X,23HNEGATIVE OF Z(J) - C(J)///48X,1HJ,10X,12H-(Z(J)-LP100880
     1C(J))//)                                                          LP100890
      DO 60 I=1, NCOL                                                   LP100900
      DO 50 J=2,MROW                                                    LP100910
      IF (I-JH(J))50,45,50                                              LP100920
   50 CONTINUE                                                          LP100930
C     WRITE OUTPUT TAPE 3,7734,I,DDT(I)                                 LP100940
 7734 FORMAT (48X,I2(9X,E13.6)                                          LP100950
      GO TO 60                                                          LP100960
   45 CONTINUE
C  45 WRITE OUTPUT TAPE 3,7735,I,DDT(I)                                 LP100970
 7735 FORMAT (48X,I2,9X,E13.6,15H (BASIS VECTOR))                       LP100980
   60 CONTINUE                                                          LP100990
C                                                                       LP101000
      K=MROW*MROW                                                       LP101010
      DO 1001 I=1,MROW                                                  LP101020
      L=0                                                               LP101030
      DO 1001 J=I,K,MROW                                                LP101040
      L=L+1                                                             LP101050
 1001 PINV(I,L)=E(J)                                                    LP101060
C     WRITE OUTPUT TAPE 3,2000                                          LP101070
 2000 FORMAT (1H1,52X,14HINVERSE MATRIX// )                             LP101080
C     DO 1010 I=2,MROW                                                  LP101090
C1010 WRITE OUTPUT TAPE 3,2001,(PINV(I,J),J=2,MROW)                     LP201095
 2001 FORMAT (1H0/(6E13.6))                                             LP101110
C                                                                       LP101200
      CALL ERROR(KOUT(1))                                               LP101210
      GO TO 2                                                           LP101220
  500 CALL ERROR (7)                                                    LP101230
      GO TO 2                                                           LP101240
      END                                                               LP101250
```

```
      SUBROUTINE ERROR(KK)                                               ERR00010
      IF (KK-4) 2,70,62                                                  ERR00020
   62 IF (KK-7) 63,72,2                                                  ERR00030
   70 WRITE OUTPUT TAPE 3,71                                             ERR00040
   71 FORMAT (21HONO FEASIBLE SOLUTION)                                  ERR00050
      GO TO 2                                                            ERR00060
   63 IF (KK-5) 2,64,67                                                  ERR00070
   64 WRITE OUTPUT TAPE 3,65                                             ERR00080
   65 FORMAT (28HONO PIVOT, INFINITE SOLUTION)                           ERR00090
      GO TO 2                                                            ERR00100
   67 WRITE OUTPUT TAPE 3,68                                             ERR00110
   68 FORMAT (26HOITERATION LIMIT EXCEEDED))                             ERR00120
      GO TO 2                                                            ERR00130
   72 WRITE OUTPUT TAPE 3,73                                             ERR00140
   73 FORMAT(23HOILLEGAL INPUT QUANTITY)                                 ERR00150
    2 RETURN                                                             ERR00160
      END                                                                ERR00170
      SUBROUTINE SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E,DOT)    MSU00920
      DIMENSION INFIX(3),A(1),B(1),TOL(4),KOUT(7),ERR(8),JH(1),X(1),     MSU00930
     1 P(1),Y(1),KB(1),E(1),ZZ(3), IOFIX(16) , TERR(8)                   MSU00940
     1,DOT(1)                                                            MSU00946
      EQUIVALENCE  (INFLAG,IOFIX(1) ), (NZ, IOFIX(2) ) ,                 MSU00950
     1            (ME,IOFIX(3) ), (MZ,IOFIX(4)), (MF,IOFIX(5)),          MSU00960
     2  (MC, IOFIX(6) ),  ( NCUT, IOFIX(7) ) , ( NVER, IOFIX(8) ),       MSU00970
     3 ( KZ,IOFIX(9) ), (ITER, IOFIX(10) ), (INVC , IOFIX(11) ) ,        MSU00980
     4 (NUMVR, IOFIX(12) ), ( NUMPV, IOFIX(13) ) ,                       MSU00990
     5  (INFS, IOFIX(14)  ) , ( JT,  IOFIX(15) ) ,( LA  , IOFIX(16) ),   MSU01000
     6 (ZZ(1),TPIV), (ZZ(2),TZERO),(ZZ(3),TCOST)                         MSU01010
      DO 1340  I= 1, 8                                                   MSU01020
       TERR(I) = 0.0                                                     MSU01030
      IOFIX(I+8)  =  0                                                   MSU01040
 1340  IOFIX(I)  =  INFIX(I)                                             MSU01050
      N = NZ                                                                01060
      M = MZ                                                                01070
      K = KZ                                                                01080
      LA   =    0                                                        MSU01090
      DO 1308  I = 1 , 3                                                 MSU01100
 1308 ZZ(I)  =  TOL(I)                                                   MSU01110
      TCOST  =  -   ABSF (TCOST)                                         MSU01120
      PMIX = PRM                                                         MSU01130
      M2 = M*M                                                           MSU01140
      INFS = 1                                                           MSU01150
      IF (N)  1304, 1304, 1371                                           MSU01160
 1371  IF (M - MF ) 1304, 1304, 1372                                     MSU01170
 1372  IF (MF - MC)  1304, 1304, 1373                                   MSU01180
 1373  IF ( MC ) 1304 , 1304,  1374                                     MSU01190
 1374  IF (ME - M ) 1304, 1375, 1375                                     MSU01200
 1375 IF( XMODF(INFLAG, 4 )  - 1 )  1400,  1320,  100                    MSU01210
```

```
1400   DO 1401 I = 1, M                                           MSU01220
1401  JH(I) = 0                                                   MSU01230
       KT   =  0                                                  MSU01240
        DO 1402  J = 1, N                                         MSU01250
      KB(J) = 0                                                   MSU01260
      MM  =  KT + MF                                              MSU01270
      LL  =  KT + M                                               MSU01280
      KQ  = 0                                                     MSU01290
      DO   1403 L = MM , LL                                       MSU01300
      IF (A(L)) 1404, 1403, 1404                                  MSU01310
1404  KQ = KQ+1                                                   MSU01320
      LQ = L                                                      MSU01330
1403    CONTINUE                                                  MSU01340
            IF (KQ - 1) 1402, 1405, 1402                          MSU01350
1405  IA = LQ- KT                                                 MSU01360
          IF ( JH(IA) ) 1402, 1406, 1402                          MSU01370
1406  IF (A(LQ)*B(IA)) 1402, 1407, 1407                           MSU01380
1407  JH(IA) = J                                                  MSU01390
      KB(J)  = IA                                                 MSU01400
1402  KT = KT + ME                                                MSU01410
1320  CONTINUE                                                    MSU01420
1100  ASSIGN  1102  TO KPIV                                       MSU01430
      ASSIGN  1114  TO KJMY                                       MSU01440
      IF (LA)  1121, 1121, 1122                                   MSU01450
1121  INVC = 0                                                    MSU01460
1122  NUMVR    =  NUMVR  +1                                       MSU01470
       DO 1101  I = 1, M2                                         MSU01480
1101  E(I)=0.                                                     MSU01490
      MM=1                                                        MSU01500
       DO 1113  I = 1, M                                          MSU01510
      E(MM) =1.0                                                  MSU01520
      X(I) = B(I)                                                 MSU01530
1113  MM = MM + M + 1                                             MSU01540
        DO 1110  I = MF, M                                        MSU01550
      IF (JH(I)) 1111, 1110, 1111                                 MSU01560
1111  JH(I) = 12345                                               MSU01570
1110  CONTINUE                                                    MSU01580
      INFS = 1                                                    MSU01590
        DO 1102  JT= 1, N                                         MSU01600
      IF ( KB(JT))  600 , 1102 , 600                              MSU01610
1114  TY = 0.                                                     MSU01620
        DO 1104  I = MF, M                                        MSU01630
      IF (JH(I)  - 12345 )  1104, 1105, 1104                      MSU01640
1105  IF (.ABSF( Y(I) ) -  TY )  1104, 1104, 1106                 MSU01650
1106  IR   = I                                                    MSU01660
      TY =  ABSF ( Y(I) )                                         MSU01670
1104    CONTINUE                                                  MSU01680
      IF  (TY - TPIV )    1107, 1108, 1108                        MSU01690
```

```
1107 KB(JT)= 0
     GO TO 1102
1108 JH(IR) = JT
     KB(JT) = IR
     GO TO 900
1102    CONTINUE
      DO 1109  I = 1, M
     IF ( JH(I) - 12345  )  1109, 1112, 1109
1112 JH(I) = 0
1109   CONTINUE
 100 ASSIGN   705  TO  NDEL
     ASSIGN  1000  TO  KJMY
     ASSIGN   221  TO  KPIV
1200   JIN  =  0
     NEG = 0
     DO 1201 I = MF, M
     IF ( ABSF ( X(I) )  - TZERO)  1202, 1203, 1203
1202 X(I) = 0.0
     GO TO 1201
1203 IF ( X(I) )  1208, 1201, 1205
1205 IF ( JH(I) )  1201, 1206, 1201
1208 NEG = 1
1206   JIN = 1
1201   CONTINUE
     IF (INFS - JIN ) 1320,  500,  200
 200 INFS = 0
 201 PMIX  =  0.0
 500 MM  = MC
 502 DO 503 J = 1, M
     P(J) = E(MM )
 503 MM  =MM  + M
     IF ( INFS )  501, 599, 501
 501 DO 504  J = 1, M
 504 P(J) = P(J)* PMIX
     DO 505  I = MF, M
     MM  =I
     IF  ( X(I) )  506, 507, 507
 506 DO 508  J = 1, M
     P(J) =  P(J) + E(MM )
 508 MM  = MM  + M
     GO TO 505
 507 IF (JH(I))  505, 509, 505
 509 DO 510  J = 1, M
     P(J) = P(J) - E(MM )
 510 MM  =  MM  +M
 505   CONTINUE
 599 CONTINUE
 700 JT = 0
```

```
      BB = TCOST                                               MSU02180
  701 DO  702  JM = 1, N                                       MSU02190
      DDT(JM)=0.                                                   2195
  703 IF ( KB(JM) )  702, 300, 702                             MSU02200
  705 IF  ( DT - BB )  708, 702, 702                           MSU02210
  708 BB = DT                                                  MSU02220
      JT = JM                                                  MSU02230
  702    CONTINUE                                              MSU02240
        IF  (JT)     203, 203, 600                             MSU02250
  203 K = 3 + INFS                                             MSU02260
      KZ = K                                                      02270
      GO TO 257                                                MSU02280
  600 DO 610  I= 1,M                                           MSU02290
  610 Y(I) =0.                                                 MSU02300
      LP  =  JT*ME - ME                                        MSU02310
      LL = 0                                                   MSU02320
      DO 605   I= 1,M                                          MSU02330
      LP = LP + 1                                              MSU02340
      IF  (A(LP))    601, 602, 601                             MSU02350
  601 DO 606  J  =   1,M                                       MSU02360
      LL = LL + 1                                              MSU02370
  606 Y(J) = Y(J) + A(LP) * E(LL)                              MSU02380
      GO TO 605                                                MSU02390
  602 LL = LL + M                                              MSU02400
  605 CONTINUE                                                 MSU02410
  699 GO TO KJMY , ( 1000 , 1114 ,  1392  )                    MSU02420
 1000 IR = 0                                                   MSU02430
      AA = 0.0                                                 MSU02440
      IA  =  0                                                 MSU02450
      DO 1050  I = MF,  M                                      MSU02460
      IF ( X(I) )  1050, 1041, 1050                            MSU02470
 1041 YI   =  ABSF ( Y(I) )                                    MSU02480
      IF   (  YI  -  TPIV   )   1050, 1050, 1042               MSU02490
 1042 IF   ( JH(I) )  1043, 1044, 1043                         MSU02500
 1043 IF   (IA)    1050, 1048, 1050                            MSU02510
 1048  IF   ( Y(I) )  1050, 1050, 1045                         MSU02520
 1044 IF    (IA)    1045, 1046, 1045                           MSU02530
 1045 IF    (   YI    -   AA    )    1050, 1050, 1047          MSU02540
 1046     IA = 1                                               MSU02550
 1047 AA   =   YI                                              MSU02560
      IR   =   I                                               MSU02570
 1050 CONTINUE                                                 MSU02580
      IF (IR)1099,1001,1099                                    MSU02590
 1001 AA = 1.0E+20                                             MSU02600
      DO 1010 IT = MF , M                                      MSU02610
      IF   ( Y(IT)  -  TPIV )  1010, 1010, 1002                MSU02620
 1002 IF   ( X(IT) ) 1010, 1010, 1003                          MSU02630
 1003 XY = X(IT) / Y(IT)                                       MSU02640
```

```
      IF ( XY - AA )  1004, 1005, 1010                                  MSU02650
 1005 IF ( JH(IT))  1010, 1004, 1010                                    MSU02660
 1004 AA = XY                                                           MSU02670
      IR = IT                                                           MSU02680
 1010 CONTINUE                                                          MSU02690
      IF (NEG)   1016, 1099, 1016                                       MSU02700
 1016 BB = - TPIV                                                       MSU02710
      DO 1030 I  = MF , M                                               MSU02720
      IF (X(I))    1012, 1030, 1030                                     MSU02730
 1012  IF   ( Y(I)     -   BB  )     1022, 1030, 1030                   MSU02740
 1022 IF   (   Y(I) * AA   -   X(I)    )    1024, 1024, 1030            MSU02750
 1024 BB   =   Y(I)                                                     MSU02760
      IR   =   I                                                        MSU02770
 1030 CONTINUE                                                          MSU02780
 1099 CONTINUE                                                          MSU02790
  206   IF( IR )     207, 207, 210                                      MSU02800
  207 K = 5                                                             MSU02810
      KZ = K                                                               02820
  257 IF (PMIX)    201, 400, 201                                        MSU02830
  210 IF (ITER -NCUT )     900,   160,   160                            MSU02840
  900 NUMPV   =   NUMPV   +   1                                         MSU02850
      YI = -Y(IR)                                                       MSU02860
      Y(IR) = -1.                                                       MSU02870
      LL = 0                                                            MSU02880
  903 DO 904  L =  IR, M2, M                                            MSU02890
      IF  ( E(L) )  905, 914, 905                                       MSU02900
  914 LL = LL + M                                                       MSU02910
      GO TO 904                                                         MSU02920
  905 XY =  E(L) / YI                                                   MSU02930
      E(L) =0.                                                          MSU02940
      DO 906  I   = 1, M                                                MSU02950
      LL= LL +1                                                         MSU02960
  906 E(LL) = E(LL) +XY* Y(I)                                           MSU02970
  904 CONTINUE                                                          MSU02980
       XY = X(IR) / YI                                                  MSU02990
      X(IR) = 0.                                                        MSU03000
      DO 908  I   = 1, M                                                MSU03010
  908 X(I) = X(I) +XY* Y(I)                                             MSU03020
      Y(IR) = -YI                                                       MSU03030
  999  GO TO KPIV , ( 221, 1102 )                                       MSU03040
  221  IA   = JH(IR)                                                    MSU03050
      IF ( IA ) 213, 213, 214                                           MSU03060
  214 KB( IA ) = 0                                                      MSU03070
  213 KB(JT)    = IR                                                    MSU03080
      JH(IR)    = JT                                                    MSU03090
      LA   =   0                                                        MSU03100
      ITER    = ITER    +1                                              MSU03110
      INVC    = INVC    +1                                              MSU03120
```

```
      IF (INVC - NVER  ) 1200, 1320,1200                                MSU03130
  160 K = 6                                                             MSU03140
      KZ = K                                                               03150
  400  ASSIGN 410 TO NDEL                                               MSU03160
      DO 401 I = 1, M                                                   MSU03170
  401 Y(I) =-B(I)                                                       MSU03180
      DO 402 I = 1, M                                                   MSU03190
      JA = JH(I)                                                        MSU03200
      IF (JA) 403, 402, 403                                             MSU03210
  403 IA =ME* (JA-1)                                                    MSU03220
      DO 405 IT = 1, M                                                  MSU03230
      IA = IA + 1                                                       MSU03240
      IF(A(IA) )  415, 405, 415                                         MSU03250
  415 Y(IT) =Y(IT) +X(I) * A(IA)                                        MSU03260
  405 CONTINUE                                                          MSU03270
  402 CONTINUE                                                          MSU03280
      DO 481  I = 1, M                                                  MSU03290
      YI  = Y(I)                                                        MSU03300
      IF ( JH(I) )  472, 471, 472                                       MSU03310
  471  YI = YI + X(I)                                                   MSU03320
  472 TERR(LA+1) = TERR(LA+1) + ABSF(YI)                                MSU03330
      IF  ( ABSF (TERR(LA+2))- ABSF (  YI   )   )   482, 481, 481       MSU03340
  482 TERR(LA+2) = YI                                                   MSU03350
  481   CONTINUE                                                        MSU03360
      DO 411  I = 1, M                                                  MSU03370
      JM = JH(I)                                                        MSU03380
      IF ( JM      )  300 , 411 , 300                                   MSU03390
  410 TERR(LA+3) -=  TERR(LA +3)  +  ABSF(DT)                           MSU03400
      IF (ABSF(TERR(LA+4))   -   ABSF(DT)  )  413, 411, 411             MSU03410
      IF (ABSF(TERR(LA+4))   -   ABSF(DT)  )  413, 411, 411             MSU03410
  413  TERR(LA+4)  =  DT                                                MSU03420
  411 CONTINUE                                                          MSU03430
      IF (LA)   193, 191,  193                                          MSU03440
  191 LA  =  4                                                          MSU03450
      IF (INFLAG  -  4  )  1320,  193,  193                             MSU03460
  193 IF  (K-5)    1392,  194,  1392                                    MSU03470
  194 ASSIGN 1392 TO KJMY                                               MSU03480
      GO TO 600                                                         MSU03490
 1304 K  =  7                                                           MSU03500
      KZ = K                                                               03510
 1392   DO  1309  I=  1, 8                                              MSU03520
 1309 ERR(I)          =   TERR(I)                                       MSU03530
       DO 1329  I = 1, 7                                                MSU03540
 1329 KOUT(I)  =  IOFIK(I+8)                                            MSU03550
      RETURN                                                            MSU03560
  300 DT = 0.                                                           MSU03570
       LL  = (JM - 1) * ME                                              MSU03580
  301 DO  303   MM  = 1, M                                              MSU03590
      LL   =  LL  + 1                                                   MSU03600
```

```
      IF ( A( LL ))304, 303, 304                                        MSU03610
  304 DT = DT + P( MM ) * A ( LL )                                      MSU03620
  303 CONTINUE                                                          MSU03630
      DDT(JM)=DT                                                            3635
  399   GO TO NDEL , ( 410 , 705 )                                      MSU03640
       END                                                              MSU03650
      END                                                                  03660
```

APPENDIX F

```
      PROGRAM LP2
      DIMENSION A(50,100),B(50),KB(100),X(5C),JH(50),P(50),Y(50),
     1E(50,50),Z(100),DDT(100),PINV(50,50),XI(50),INFIX(10),TOL(10),
     2KOUT(10),ERR(10),RUN(8),QUOTA(50),PROB(5C),AV(50),ANS(50),
     3BSTORE(5C,52),CLOSS(5C),ZBAR(50),PERCT(50)
     OCOMMON TCL,R,Y,E,Z,CDT,PINV,XI,A,B,KB,X,JH,P,INFIX,KCUT,ERR,RUN,
     1NCLL,MRWW,MROW1,MAX,TIME,PROP,SUMANS,BSTORE,AMAX,MAX1,MAX2,PRM,
     2CDT,NCCL,MROW,PERCT,QUOTA,PROB,AV,ANS,BSTORE,CLOSS,VARCST
      EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MRCW),(NRUN,RUN(1))
C
C     A MODIFICATICN OF LP1 TO GENERATE A TRANSPCRTATICN MATRIX          LP200012
C                                                                        LP200011
    2 DO 10 I= 1, 100
      DO 10 J=1,50
   10 A(I,J)=0.
C
C     PERMANENT DATA
C        INFIX (3)=MAX NO. OF RCWS (CF. DIMENSION STATEMENT)             LP100100
C        INFIX(7)=MAX INTERATION CCUNT
C
      B(1)=C.
      INFIX(1)=4
      INFIX(3)=50
      INFIX(5)=2
      INFIX(6)=1
      INFIX(7)=10C                                                       LP100190
      INFIX(8)=0                                                         LP100200
      TOL(1)=1.E-7                                                       LP10021C
       TOL(2) = 1.E-5                                                    LP100220
      TOL(3) = -1.E-6                                                    LP100230
      TOL(4) = 1.E-7                                                     LP100240
       PRM = 0.                                                          LP100250
C                                                                        LP100260
C     INPUT --  NOTE.. MROW = NO. OF ROWS PLUS CNE, BECAUSE COST         LP100270
C                      COEFFICIENTS ARE ENTERED AS RCW CNE.              LP10028C
C                                                                        LP100290
      READ INPUT TAPE 2,3,NCLL,MRWW,RUN                                  LP200300
    3 FORMAT (2I5,7A8,A6)                                                LP100310
      IF (NRUN-4HSTOP)9999,9998,9999                                     LP200301
 9998 WRITE CUTPUT TAPE 3,9997                                           LP200302
 9997 FORMAT (1H1,10X,12HJCB COMPLETE)                                   LP200303
      STOP                                                               LP200304
 9999 NCOL=NCLL*MRWW                                                     LP20C305
      IF (NCOL) 5C0,500,8                                                LP100320
    8 MROW=NCLL+MRWW                                                     LP200325
      MRWW1=MRWW+1                                                       LP200330
      MRWW2=MRWW+2                                                       LP20C335
```

```
      5 READ INPUT TAPE 2,4,(A(1,J),J=1,NCOL)                            LP200340
C                                                                        LP200345
        MROW1= MROW-1                                                    LP200326
C                     NOTE- THE A(1,J) ROW ARE THE COST CCEFFICIENTS     LP100350
C         THE C-I,J-S ARE READ IN ROW-WISE                               LP200355
C                                                                        LP200356
        READ INPUT TAPE 2,7733,MAX,PRCP,TIME
   7733 FORMAT (I10,2F10.5)
      4 FORMAT (7F10.5)                                                  LP100360
        J=0                                                              LP200361
        DO 4000 I=2,MRWW1                                                LP200362
        DO 4000 K=1,NCLL                                                 LP200363
        J=J+1                                                            LP200364
   4000 A(I,J)=1..0                                                      LP200365
        J=0                                                              LP200366
        DO 4001 I=MRWW2,MROW                                             LP200367
        J=J+1                                                            LP200368
        DO 4001 K=J,NCOL,NCLL                                            LP200369
   4001 A(I,K)=1.0                                                       LP200370
C
        MAX1=MAX+1
        AMAX=MAX
        DO 8000 M=1,MAX2
        DO 8000 J=2,MROW
   8000 BSTORE(J,M)=0.0
        VARCST=0.0
        SUMANS=0.0                                                       PROB 103
        WRITE OUTPUT TAPE 3,29,RUN                                       LP100620
     29 FORMAT(1H1,36X,7A8,A6)
        ITIME=XINTF(TIME)
        WRITE OUTPUT TAPE 3,7734,MAX
   7734 FORMAT(///,48X,17HNUMBER OF SAMPLES,I5)
        WRITE OUTPUT TAPE 3,7770
   7770 FORMAT(///,43X,34HROUTE NUMBERS AND ASSOCIATED COSTS ///)
        WRITE OUTPUT TAPE 3,7774,((J,A(1,J)), J=1,NCCL)
   7774 FORMAT(13X,6(I3,F8.2,5X)/)
        READ INPUT TAPE 2,7,( QUOTA(I), I=2, MROW1)                      PROB   6
      7 FORMAT (7(F8.0,2X))                                              TEST  70
        READ INPUT TAPE 2,9,(PROB(I), AV(I), I=2, MROW1)                 PROB 107
      9 FORMAT (3(F10.5,F10.0))                                          TEST 720
        WRITE OUTPUT TAPE 3,7775,ITIME,PROP
   7775 FORMAT(1H1,40X,35HNUMBER OF RECRUITING DAYS IN PERIOD,I5 ///
       143X,19HGREAT LAKES PERCENT ,3X,F10.3)
        WRITE OUTPUT TAPE 3,7735
   7735 FORMAT(///17X,14HSTATION NUMBER,7X,5HQUOTA,7X,24HAVERAGE INTERVIEW
       1 NUMBER,5X,25HPROB OF INTERVIEW SUCCESS//)
        DO 7736 J=2,MROW1
        I=J-1
```

```
      WRITE OUTPUT TAPE 3,7736,I,QUOTA(J),AV(J),PROB(J)
 7736 FORMAT(23X,I2,5X,F13.0,9X,F13.0,17X,F10.3)
      TOTAL=0.0
      DO 7737 J=2,MROW1
 7737 TOTAL=TOTAL+QUOTA(J)
      PRINT 7738,TOTAL
 7738 FORMAT(/25X,5HTOTAL,F13.0)
      DO 6666 M=1,MAX                                                   PROB0135
      CALL RORLIT(M)
C ******  NOTE-THE ORIGIN INFORMATION IS READ FIRST, FOLLOWED BY THE    LP200374
C              DESTINATION INFORMATION. THE LAST DESTINATION IS EXCLUDED. LP200375
C                                                                       LP200376
C                                                                       LP10059C
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E,DOT)         LP100600
C                                                                       LP100610
      WRITE OUTPUT TAPE 3,30,Y(1)
   30 FORMAT(1H1,///29X,37HMINIMUM COST OF ALL TRANSPORTATION IS 2X,F13.
     12,////43X,23HROUTES USED AND AMOUNTS ///36X,7HROUTES ,16X,18HNUMBE
     2R MEN SHIPPED //)
C                                                                       LP100830
   40 WRITE OUTPUT TAPE 3,43,(JH(I),X(I),I=2,MROW)                      LP100840
   43 FORMAT(39X,     I2,     19X,F13.0)
C                                                                       LP101000
      K=MROW*MROW                                                       LP101010
      DO 1001 I=1,MROW                                                  LP101020
      L=0                                                               LP101030
      DO 1001 J=I,K,MROW                                                LP101040
      L=L+1                                                             LP101050
 1001 PINV(I,L)=E(J)                                                    LP101060
      CALL ERROR(KOUT(1))                                               LP101210
      GO TO 8325                                                        LP101220
  500 CALL ERROR (7)                                                    LP101230
      GO TO 8325                                                        LP101240
 8325 CALL AVERAG(M)
 6666 CONTINUE
      CALL RISK
      END                                                               LP101250
C                                                                       LP100660
      SUBROUTINE ERROR(KK)                                              ERR00010
      IF (KK-4) 2,70,62                                                 ERR00020
   62 IF (KK-7) 63,72,2                                                 ERR00030
   70 WRITE OUTPUT TAPE 3,71                                            ERR00040
   71 FORMAT (21HONO FEASIBLE SOLUTION)                                 ERR00050
      GO TO 2                                                           ERR00060
   63 IF (KK-5) 2,64,67                                                 ERR00070
   64 WRITE OUTPUT TAPE 3,65                                            ERR00080
   65 FORMAT (28HONO PIVOT, INFINITE SOLUTION)                          ERR00090
      GO TO 2                                                           ERR00100
```

```
   67 WRITE CUTPUT TAPE 3,68                                          ERR00110
   68 FORMAT (26HOITERATICN LIMIT EXCEEDED) )                         ERR0012C
      GO TO 2                                                         ERR0013C
   72 WRITE OUTPUT TAPE 3,73                                          ERR0C140
   73 FORMAT(23HOILLEGAL INPUT QUANTITY)                              ERR00150
    2 RETURN                                                          ERR0016C
      END                                                             MSU03650
C
      SUBROUTINE RCRUIT(MIKE)
     0DIMENSION A(5C,100),B(50),KB(1CO),X(50),JH(5C),P(5C),Y(5C),
     1E(50,5C),Z(100),DDT(100),PINV(50,50),XI(50),INFIX(10),TOL(10),
     2KOUT(10),ERR(10),RUN(8),QUOTA(5C),PROB(50),AV(5C),ANS(50),
     3BSTORE(50,52),CLOSS(50),ZBAR(5C),PERCT(50)
     0COMMON TCL,R,Y,E,Z,DDT,PINV,XI,A,B,KB,X,JH,P,INFIX,KOUT,ERR,RUN,
     1NCLL,MRWW,MROW1,MAX,TIME,PROP,SUMANS,BSTORE,AMAX,MAX1,MAX2,PRM,
     2DDT,NCOL,MROW,PERCT,QUOTA,PROB,AV,ANS,BSTORE,CLOSS,VARCST
      EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MROW),(NRUN,RUN(1))
      WRITE OUTPUT TAPE 3,7776,MIKE
 7776 FORMAT(1H1,///45X,6HSAMPLE,I4///20X,14HSTATION NUMBER ,6X,19HNUMBE
     1R MEN ENLISTED ,5X,20HPERCENT QUOTA FILLED ///)
      DO 5112 J=2,MROW1                                               TEST 150
      OUTLIM=POISON(AV(J),TIME)                                       PROB  16
 5109 REC=DIST(OUTLIM,PROB(J))                                        PROB 130
 5004 K=J-1                                                           PROB 230
      IF (REC - OUTLIM) 5008,5005,5005                                PROB  26
 5005 IF (OUTLIM - QUOTA(J)) 5007,5007,5010                           PRCB  27
 5007 B(J) = OUTLIM                                                   PROB  28
      GO TO 5011
 5008 IF (REC - QUOTA(J)) 5009,5009,5010                              PROB030
 5009 B(J)=REC                                                        PROB0031
      GO TO 5011
 5010 B(J)=QUOTA(J)                                                   PROB0033
 5011 CONTINUE
      PERCT(J)=B(J)/QUOTA(J)*100.                                     PROB 340
      WRITE OUTPUT TAPE 3,7778,K,B(J),PERCT(J)                        TEST1330
 7778 FORMAT(26X,I2,11X,F13.0,15X,F13.3)
 5112 CONTINUE                                                        TEST 340
 5013 I=2                                                             PRCB0035
      SUM=0.0                                                         PROB0036
 5014 SUM=SUM+B(I)                                                    PRCB0037
      IF (I-(MROW-1)) 5015,5016,5016                                  PROB038
 5015 I=I+1                                                           PROB0039
      GO TO 5014                                                      PROB0040
 5016 HOLD = INTF(SUM * PROP)                                         PROB  41
      IF ((HOLD - (SUM * PROP)) - 0.5) 5026,5027,5027                 PROB 142
 5026 B(MROW) = HOLD                                                  PROB 143
      GO TO 5028                                                      PROB 144
 5027 B(MROW) =  HOLD + 1.                                            PROB 145
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```
 5028 CONTINUE                                                              PROB 145
      SUMQTA=0.0
      DO 5029 J=2,MROW
      BSTORE(J,MIKE)=B(J)
      BSTORE(J,MAX1)=BSTORE(J,MAX1)+B(J)
 5029 SUMQTA=SUMQTA+QUOTA(J)
      PERCT(MROW)=B(MROW)/(SUMQTA*PROP)*100.
      WRITE OUTPUT TAPE 3,7779,MROW1,B(MROW),PERCT(MROW)
 7779 FORMAT(26X,I2,11X,F13.0,15X,F13.3 )
      RETURN
      END
C
      FUNCTION POISON(AVG,TIME)                                             PROB 690
C     THIS FUNCTION GENERATES A NORMAL APPROXIMATION TO POISSON ARRIVALSPROB 705
C                                                                           PROB 710
      ALPHA=AVG*TIME                                                        PROB 715
      GUESS=RDUN(DUMMY)                                                     PROB 720
      RAND=RDUN(DUMMY)                                                      PROB 730
      UONE=1./RAND                                                          PROB 735
      XONE=SQRTF(2.*LOGF(UONE))*COSF(6.2831853*GUESS)                       PROB 740
 7001 POISON=INTF(XONE*SQRTF(ALPHA)+ALPHA-.5)                               PROB 790
      IF (POISON) 7012,7002,7002
 7012 POISON=0.0
 7002 RETURN                                                                PROB 800
      END                                                                   PROB 810
C
      FUNCTION DIST(TRIALS,PARAM)                                           PROB  58
      K=XINTF(TRIALS)                                                       PROB 158
      DIST=0.0                                                              PROB0059
      IF (K) 5024,5024,5025                                                 PROB 159
 5025 DO 5023 L=1,K                                                         PROB 261
      RAND=RDUN(DUMMY)                                                      PROB0061
      IF(PARAM-RAND) 5023,5022,5022                                         PROB0062
 5022 DIST=DIST+1.                                                          PROB0063
 5023 CONTINUE                                                              PROB0064
 5024 RETURN                                                                PROB  65
      END                                                                    PROB 66
C                                                                            PROB 67
      FUNCTION RDUN(DUMMY)                                                  RDUN000
C     RDUN(DUMMY) IS THE RANDOM NUMBER GENERATOR                             PROB 68
C                                                                           LP200011
      CON(R1=0B,K1=1220703125,K2=20000000000000000B,R=0000000077777777B)   RDUN 100
   10 LDA(R),MUF(K1),DVF(K2),STQ(R)                                         RDUN0200
   11 LLS(48),DVF(K2),STA(R2),ARS(11)                                       RDUN0300
   12 ADD(K2),FAD(K2)                                                       RDUN0400
   13 RETURN                                                                RDUN0500
      END                                                                   ERR00170
C
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      SUBROUTINE AVERAG(MIKE)
     0DIMENSION A(50,100),B(50),KB(100),X(50),JH(50),P(50),Y(50),
     1E(50,50),Z(100),DDT(100),PINV(50,50),XI(50),INFIX(10),TOL(10),
     2KOUT(10),ERR(10),RUN(8),QUOTA(50),PROB(50),AV(50),ANS(50),
     3BSTORE(50,52),CLOSS(50),ZBAR(50),PERCT(50)
     0COMMON TOL,R,Y,E,Z,DDT,PINV,XI,A,B,KB,X,JH,P,INFIX,KCUT,ERR,RUN,
     1NCLL,MRWW,MROW1,MAX,TIME,PROP,SUMANS,BSTORE,AMAX,MAX1,MAX2,PRM,
     2DDT,NCOL,MROW,PERCT,QUOTA,PROB,AV,ANS,BSTORE,CLOSS,VARCST
      EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MROW),(NRUN,RUN(1))
      AMIKE=MIKE
      ANS(MIKE)=Y(1)
      SUMANS=SUMANS+ANS(MIKE)
      VARCST=VARCST+ANS(MIKE)*ANS(MIKE)
      AVCOST=SUMANS/AMIKE
      IF (MIKE-1) 5066,5066,5065
 5065 DEVCST=(VARCST-AMIKE*AVCOST*AVCOST)/(AMIKE-1.)
      DEVCST=SQRTF(DEVCST)
      GO TO 5067
 5066 DEVCST=0.0
 5067 PRINT 5057,AVCOST,DEVCST
 5057 FORMAT(//34X,12HAVERAGE COST,14X,21HSTD DEVIATION OF COST//32X,
     1F13.2,16X,F13.2//)
      RETURN
      END
C                                                                       PROB 710
      SUBROUTINE RISK
     0DIMENSION A(50,100),B(50),KB(100),X(50),JH(50),P(50),Y(50),
     1E(50,50),Z(100),DDT(100),PINV(50,50),XI(50),INFIX(10),TOL(10),
     2KOUT(10),ERR(10),RUN(8),QUOTA(50),PROB(50),AV(50),ANS(50),
     3BSTORE(50,52),CLOSS(50),ZBAR(50),PERCT(50)
     0COMMON TOL,R,Y,E,Z,DDT,PINV,XI,A,B,KB,X,JH,P,INFIX,KCUT,ERR,RUN,
     1NCLL,MRWW,MROW1,MAX,TIME,PROP,SUMANS,BSTORE,AMAX,MAX1,MAX2,PRM,
     2DDT,NCOL,MROW,PERCT,QUOTA,PROB,AV,ANS,BSTORE,CLOSS,VARCST
      EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MROW),(NRUN,RUN(1))
      VARSK=0.0
      TLOSS=0.0
      DO 5052 J=2,MROW
 5052 BSTORE(J,MAX1)=BSTORE(J,MAX1)/AMAX
      WRITE OUTPUT TAPE 3,5070
 5070      FORMAT(1H1,///,33X,14HSTATION NUMBER ,11X,20HMEAN NUMBER ENLI
     1STED ,///)
      DO 5071 J=2,MROW
      K=J-1
      WRITE OUTPUT TAPE 3,5071,K,B(J)
 5071 FORMAT (39XI2,14X,F13.0)
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E,DDT)
      K=MROW*MROW                                                       LP101010
      DO 1001 I=1,MROW                                                  LP101020
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      L=0                                                               LP101030
      DO 1001 J=I,K,MROW                                                LP101040
      L=L+1                                                             LP101050
 1001 PINV(I,L)=E(J)                                                    LP101060
      WRITE OUTPUT TAPE 3,39,Y(1)
   39 FORMAT(1H1,29X,48H MINIMUM COST OF ALL TRANSPORTATION OF MEANS IS
     1      F13.2////
     243X,23HROUTES USED AND AMOUNTS   ///36X,7HROUTES ,16X,18HNUMBER MEN
     3 SHIPPED  //)
      WRITE OUTPUT TAPE 3,5061,(JH(I),X(I),I=2,MROW)
 5061 FORMAT(39X,I2,19X,F13.0)
      DO 5062 M=1,MAX1
 5062 ZBAR(M)=0.0
      WRITE OUTPUT TAPE 3,5086
 5086 FORMAT(1H1,/////30X,6HSAMPLE ,5X,23HCOST USING RANDOM INPUT ,5X,
     119HLOSS ON THIS SAMPLE //)
      DO 5058 M=1,MAX
      DO 5080 K=2,MROW
 5080 B(K)=BSTORE(K,M)
      DO 5051 I=2,MROW
      X(I)=0.0
      DO 5051 K=2,MROW
 5051 X(I)=X(I)+PINV(I,K)*B(K)
      DO 5050 I=2,MROW
      IF (X(I)) 5053,5050,5050
 5053 IF (XMODF(JH(I),2)) 5054,5054,5056
 5054 DO 5055 L=2,MROW
      IF (JH(I)-JH(L)-1) 5055,5063,5055
 5055 CONTINUE
      PRINT 5090
 5090 FORMAT(7HERROR 1)
      GO TO 5050
 5056 DO 5065 L=2,MROW
      IF (JH(L)-JH(I)-1) 5065,5063,5065
 5065 CONTINUE
      PRINT 5095
 5095 FORMAT(7HERROR 2)
      GO TO 5050
 5063 X(L)=X(L)+X(I)
      X(I)=0.0
      PRINT 5060,M
 5060 FORMAT(/10X,29HINFEASIBLE SOLUTION ON SAMPLE ,I2//)
 5050 CONTINUE
      DO 5064 I=2,MROW
      K=JH(I)
      ZBAR(M)=ZBAR(M)+X(I)*A(1,K)
 5064 CONTINUE
      CLOSS(M)=ZBAR(M)-ANS(M)
```

```
      TLOSS=TLCSS+CLOSS(M)
      VARSK=VARSK+CLOSS(M)*CLOSS(M)
      WRITE OUTPUT TAPE 3,5058,M,ZBAR(M),CLOSS(M)
 5058 FORMAT(31X,I3,10X,F13.2,12X,F13.2)
      AVLOSS=TLOSS/AMAX
      VARSK=(VARSK-AMAX*AVLOSS*AVLOSS)/(AMAX-1.)
      DEVRSK=SGRTF(VARSK)
      WRITE CUTPUT TAPE 3,5059,AVLCSS,DEVRSK
 5059 FORMAT (//30X,4HRISK              ,F13.2,10X,25HSTD DEVIATICN CF LCSS
     1    ,F13.2 //////)
      RETURN
      END
C
```